An Elasto-Plastic Model for Analysis of Underwater Tunnels, Excavated in a Strain-Softening Hoek-Brown Rock Mass

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Abstract

In this paper, an analytical method for the analysis of underwater tunnels in axisymmetric plain strain conditions is presented. A new technique has been proposed using finite difference numerical method for calculating the distribution of pore water pressure as well as stress and strain around a circular tunnel excavated in rock mass. Behavior of the rock mass around the tunnel is considered elasto-plastic in strain-softening model. Meanwhile, the effects of increment in elastic strain within plastic zone and dilatancy angle are taken into account. Seepage flow and secondary permeability due to hydraulic-mechanical coupling have also been considered in the plastic zone.

A more accurate model has been provided for calculating the distribution of pore pressure in the elastic zone by modifying the previous models. Since governing equations do not have a closed form solution, a computer program has been prepared based on the proposed model. Accuracy and applicability of this method have been investigated with an example.

Key words: Underwater Tunnels, Seepage, Pore Pressure Distribution, Groundwater, Strain-Softening

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1. Introduction

When a tunnel is excavated below groundwater table, groundwater flows into the tunnel and seepage forces act on the tunnel wall. Any element of rock mass is loaded on all side by seepage forces as body forces. Excavation of tunnel will affect permeability of the surrounding rock mass. These changes in the permeability can be due to state of stress,

pore pressure and plastic deformations. Some cracks may be formed in the fractured rock mass by increasing the stress, while permeability of the rock mass is changed due to widening of cracks and their increased spacing.
Fields of stress and deformation created by tunnel excavation and seepage flow in the underwater tunnels have been studied by numerous researchers. Brown and Bray (Brown & Bray, 1982) examined hydraulic-mechanical coupling in rock mass for analysis of underwater tunnels considering the changes of rock permeability in plastic zone. However, they failed to provide an accurate model for calculation of seepage in their equations. Fahimifar (Fahimifar & Zareifard, 2009) made their analytical model taking into account the hydraulic-mechanical coupling of rock mass and lining, considering effective stresses rather than total stresses and modifying the accurate seepage model of Kolymbas (Kolymbas & Wagner, 2007). The elastic strain has been assumed constant within plastic zone by these methods with the effect of elastic strain being ignored in the plastic zone, such that the increment of plastic strain has been considered versus total strain. Moreover, the dilatancy angle has been taken constant in the strain-softening zone with its effects on deformations around the tunnel being disregard.
An analytical model has been proposed in this study based on the method of Brown and Bray along with development of the accurate seepage model of Ming (Ming, Meng-Shu, Tan, & Xiu-Ying, 2010). Variations of the dilatancy angle, increment of elastic strain in plastic zone and deviatoric plastic strain as the parameter of softening have been also taken into account in this method.

2. Model assumptions and governing equations

The computational model is considered by assuming axisymmetric condition. This model involves different zones of the rock mass including elastic and plastic zones (the zone with strain-softening and the zone with residual strength).
Equilibrium equation in the axisymmetric conditions for each element of rock mass in polar coordinates will be stated as Eq. 1 (Timoshenko & Goodier, 1994):

$$\frac{d\sigma_r}{dr} - \frac{(\sigma_\theta - \sigma_r)}{r} = 0 \tag{1}$$

In the axisymmetric conditions, equation of deformation-strain will be defined as below:

$$\varepsilon_r = -\frac{du}{dr} \tag{2}$$

$$\varepsilon_\theta = \frac{-u}{r} \tag{3}$$

$$\frac{d\varepsilon_r}{dr} = \frac{\varepsilon_r - \varepsilon_\theta}{r} \tag{4}$$

In Eq. 1, σ_θ and σ_r denote major and minor principle stresses, respectively. In Eqs. 2~4, ε_r and ε_θ denote radial and tangential strains respectively, while u is radial deformation.

3. Rock mass Behavior

Nonlinear Hoek-Brown empirical strength criterion (Hoek & Brown, 1980) is used for a rock mass. For a rock mass:

$$\sigma_\theta^{'} - \sigma_r^{'} = \left\{m\sigma_r^{'}\sigma_c + s\sigma_c^2\right\}^{\frac{1}{2}} \tag{5}$$

And for broken or plastic zone:

$$\sigma_\theta^{'} - \sigma_r^{'} = \left\{m_r\sigma_r^{'}\sigma_c + s_r\sigma_c^2\right\}^{\frac{1}{2}} \tag{6}$$

In the equations above, σ_θ' and σ_r' show major and minor effective stresses at the failure respectively; σ_c is uniaxial compressive strength of the intact rock material, m_r , s_r , m and s are strength parameters of intact and broken rock mass respectively.
The behavioral model of rock mass is the strain--softening model. The rock mass will show an elastic behavior as long as the equation of principle stresses does not satisfy strength criterion. Thereafter, the strength of rock mass will gradually incline to residual strength. Parameters of $\varphi(i)$, $\sigma_c(i)$, $m(i)$ and $s(i)$ are stated in terms of γ^p function (deviatoric plastic strain). In the plastic zone, it is assumed that the mentioned parameters can be described as a bilinear function of deviatoric plastic strain (γ^p) (Alonso, Alejano, Varas, Fdez-Manin, & Carranza-Torres, 2003).

$$w(i) = \begin{cases} w_p - (w_p - w_r)\dfrac{\gamma^p(i)}{\gamma^{p*}} & 0 < \gamma^p(i) < \gamma^{p*} \\ w_r & \gamma^p(i) > \gamma^{p*} \end{cases} \tag{7}$$

It should be noted in this model that γ^p is strain-softening parameter for controlling strength parameters of $\varphi(i)$, $\sigma_c(i)$, $m(i)$ and $s(i)$ in the strain-softening zone and is defined according to Eq. 8 (Park, Tontavanich, & Lee, 2008):

$$\gamma^p = \varepsilon_\theta^p - \varepsilon_r^p \tag{8}$$

Where, ε_θ^p and ε_r^p denote tangential and radial plastic strains, respectively.

4. Hydraulic Analysis

4.1. Permeability of Rock Mass

Permeability is correlated with deformations as a result of these hydraulic-mechanical couplings in the rock mass. In this regard, Eq. 9 has been represented (Ghadami, 2012):

$$K_r = K_o(1 + \eta\varepsilon_{vp}^2) \tag{9}$$

In the Eq. 9, η is the parameter which represents permeability variations of rock mass in plastic zone, K_o is initial permeability of rock mass and is ε_{vp} volumetric plastic strain $(\varepsilon_{vp} = \varepsilon_{rp} + \varepsilon_{\theta p})$.

4.2. Seepage Pattern and Pore Pressure distribution

Patterns represented in this study for seepage in underwater tunnels in depicted in Fig. 1 (Radial convergent seepage is considered for the plastic zone in the proposed model).
The equations provided for the seepage patterns are based on the following assumptions:

- Permeability of rock mass is homogenous and isotropic.
- The flow is in steady state.
- Tunnel has a circular cross section with constant hydraulic potential.

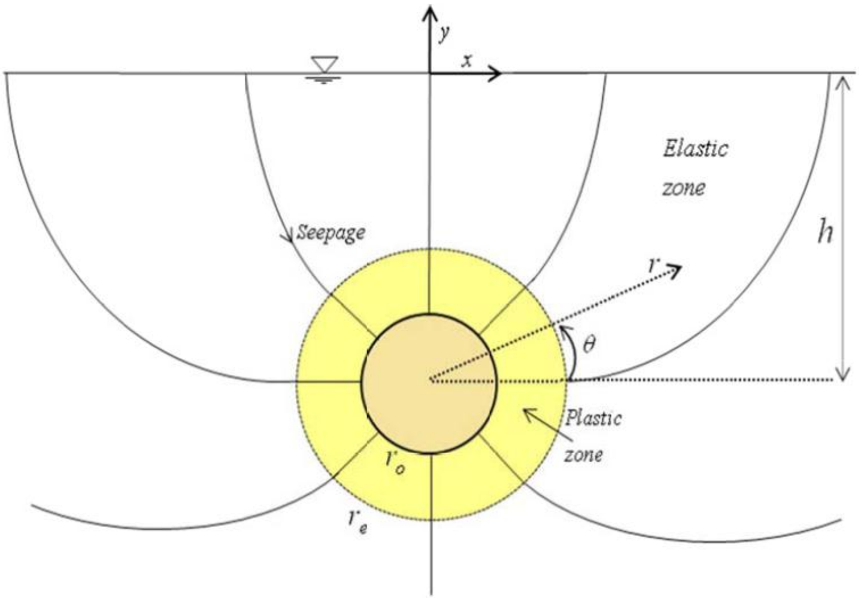

Fig 1: Seepage pattern in underwater tunnel

In this study, a more accurate model has been produced for calculation of pore pressure by modifying the model offered by Ming (Ming, Meng-Shu, Tan, & Xiu-Ying, 2010) in order to analyze the seepage in underwater tunnels at elastic zone. Furthermore, Darcy's law for radial seepage is used for analysis of the seepage at plastic zone.

4.3. Elastic Zone

The equation suggested by Ming et al. is generally given by Eq. 10 for the case when pore pressure is constant at outer surface of the tunnel (Ming, Meng-Shu, Tan, & Xiu-Ying, 2010):

$$P_w(x,y) = X(x,y) + \frac{P_a + Y(x,y)}{\ln\left[\frac{h}{r_o^2} - \sqrt{(\frac{h}{r_o^2})^2 - 1}\right]}\left(\ln\frac{x^2 + (y+\sqrt{h^2-r_o^2})^2}{x^2 + (y-\sqrt{h^2-r_o^2})^2}\right) \tag{10}$$

Where, r_o is outer radius of the tunnel, h represents depth of the tunnel from groundwater level, P_a denotes pore pressure at outer surface of the tunnel, while $X(x,y)$ and $Y(x,y)$ are functions which are determined according to boundary conditions in underwater tunnels.

Since the above equation is capable of calculating seepage in different directions, replacing Cartesian coordinates with polar coordinates and applying boundary conditions, one can obtain the equation for distribution of pore pressure as (Ghadami, 2012):

$$P_w(r,\theta) = (h - r\sin\theta)\gamma_w + \frac{P_a - (h - r_o\sin\theta)\gamma_w}{\ln\left[\frac{(r_o\cos)^2 + (r_o\sin\theta - h + \sqrt{h^2 - r_o^2})^2}{(r_o\cos)^2 + (r_o\sin\theta - h - \sqrt{h^2 - r_o^2})^2}\right]}\left(\ln\frac{(r\cos\theta)^2 + (r\sin\theta - h + \sqrt{h^2-r_o^2})^2}{(r\cos\theta)^2 + (r\sin\theta - h - \sqrt{h^2-r_o^2})^2}\right) \tag{11}$$

4.4. Plastic Zone

In the plastic zone around tunnel, assuming a radial flow, distribution of pore pressure is obtained by using Darcy's low (Fahimifar & Zareifard, 2009):

$$P_w(r,\theta) = \gamma_w q / 2\pi \int_{r_o}^{r} \frac{1}{rK_r(r)} dr + P_a - \gamma_w (r - r_o)\sin\theta \tag{12}$$

Where, is permeability of rock mass in plastic zone and q is seepage rate.

Equating the values of pore pressure at elasto-plastic interface related to the elastic zone in Eq. 11 with those in Eq. 12 related to the plastic zone will lead to find the value of flow rate.

5. Stresses and Deformations of Rock Mass

By using equation 1 and 5, the equilibrium equation for the plastic zone is written as below:

$$\frac{d\sigma_r}{dr} = \frac{\sigma_\theta - \sigma_r}{r} = \frac{\left[m(\sigma_r - P_w)\sigma_c + s\sigma_c^2\right]^{\frac{1}{2}}}{r} \tag{13}$$

Since Eq. 13 does not have a closed form solution, radial and tangential stresses are obtained at each step using a numerical solution (finite difference method) (Brown & Bray, 1982)

$$\sigma_r(i) = b - \sqrt{b^2 - a} \tag{14}$$

$$\sigma_\theta(i) = \sigma_r(i) + \left[\bar{m}(i)\overline{\sigma_c}(i)\sigma_r(i) + \bar{s}(i)\overline{\sigma_c}^2(i)\right]^{\frac{1}{2}} \tag{15}$$

Where:

$$a = \sigma_r^2(i-1) - 4c\left[\frac{1}{2}\bar{m}(i)\overline{\sigma_c}(i)(\sigma_r(i-1)) - P_w(i) - P_w(i-1)) + \bar{s}(i)\overline{\sigma_c}^2(i)\right]$$

$$b = \sigma_r(i-1) + c\overline{m_a}(i)\overline{\sigma_c}(i)$$

$$c = \left[\frac{r_{i-1} - r_i}{r_{i-1} + r_i}\right]^2 \tag{16}$$

$$\bar{w}(i) = \frac{1}{2}(w(i-1) + w(i))$$

In Eq. 16, w represents each of the strength parameters of s, m, σ_c and φ.

In contrast with Brown-Bray method, which used to consider the elastic strain constant within the whole plastic zone, elastic strain increment is separately considered at each step of calculation by the proposed model. Thus, total strain would be divided into two parts, namely elastic and plastic strains.

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \end{Bmatrix} = \begin{Bmatrix} \varepsilon_r^e \\ \varepsilon_\theta^e \end{Bmatrix} + \begin{Bmatrix} \varepsilon_r^p \\ \varepsilon_\theta^p \end{Bmatrix} \tag{17}$$

In the proposed model, the correlation between radial plastic strain increment $(\Delta\varepsilon_r^p(i))$ and tangential plastic increment $(\Delta\varepsilon_\theta^p(i))$ is given by Eq. 18 (Wang, 1996).

$$\Delta\varepsilon_r^p(i) = -K(i)\Delta\varepsilon_\theta^p(i) \tag{18}$$

$$K(i) = \frac{1+\sin\varphi(i)}{1-\sin\varphi(i)} \tag{19}$$

Where, φ denotes dilatancy angle.

One solution runs the calculations at elasto-plastic interface assuming an elasto-plastic radius. Then it numerically solves the equations of plastic zone considering the values of stress and strain obtained at the elasto-plastic interface as initial values as long as the boundary conditions are met. The calculations are followed up until elasto-plastic radius reaches a constant value.

6. Verification of the Proposed Model

Since the proposed model does not have a closed form solution, Utunnel (Underwater Tunnel) program is programmed by MATLAB software. This program is analyzed using the sample tunnel, while its obtained results have been interpreted and then compared with those from other models in order to verify it.
A tunnel has been excavated rock masses with parameters listed in table 1.

Parameter	Value	Parameter	Value
Young Modulus (MPa) (E)	2000	m_P	0.65
Poisson ratio (υ)	0.2	s_P	0.2
Initial stress (MPa) (P_o)	27	m_r	0.2
Tunnel radius (m) (r_o)	3.0	s_r	0.0001
σ_c (MPa)	40	Internal friction angle (ϕ_P)	30
Height of groundwater (m) (h)	300	Strain-softening parameter (γ^*)	0.00375
Rock permeability (m/s) (K_o)	10^(-6)		

Table 1.Data of the tunnel analyzed by Brown-Bray method

With respect to these specifications of table 1, Brown and Bray analyzed this tunnel and cited the obtained results accordingly. Their results were compared with those of Utunnel program in table 2. Fig. 6 depicts the diagrams of ground response curve and those of σ_θ and σ_r versus radius r calculated by Brown method (Brown & Bray, 1982) and Fahimifar method (Fahimifar & Zareifard, 2009), compared with those extracted by Utunnel program.

Parameter	Brown & Bray model	Utunnel program		
		$\varphi = 0$	$\varphi = \phi/4$	$\varphi = \phi/2$
Elasto-plastic radius (m)	16.024	16.5148	17.1237	17.7426
Radial stress in elasto-plastic radius (MPa)	16.73	16.8073	16.8122	16.8169
Tangential stress in elasto-plastic radius (MPa)	36.9	37.1927	37.1878	37.1831
Ground displacement at tunnel radius (m)	0.1434	0.0881	0.149	0.302

Table 2. Comparison of the results of Utunnel program with those of Brown-Bray method

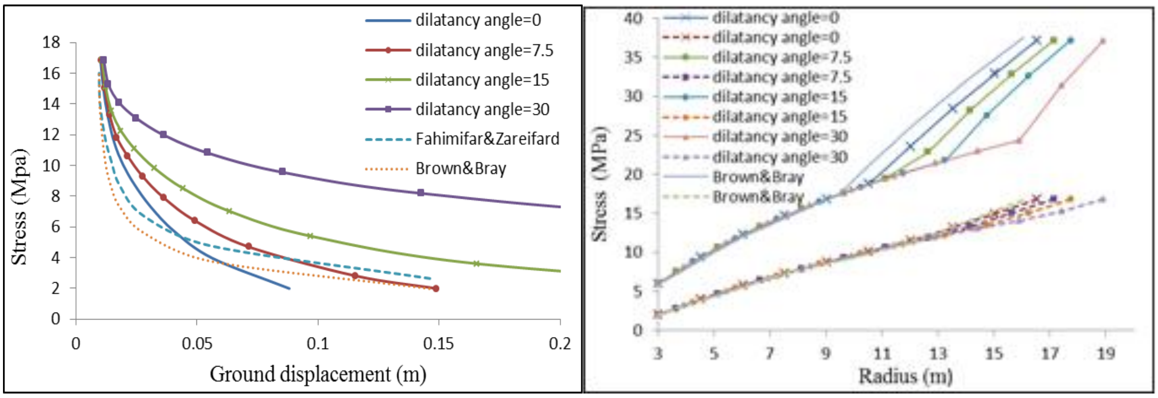

Fig 2: Ground response curve (left), radial and tangential stresses at plastic zone (Plain lines denote tangential stress while dotted lines represent radial stress) (right), h=300 m

The effects of changing dilatancy angle and elastic strain increment have not been considered at the plastic zone. Furthermore, Brown and Bray have utilized the inaccurate radial seepage pattern for hydraulic analysis. In spite of using the accurate seepage model of Kolymbas and Wagner (2007), the effects of changing dilatancy angle and elastic strain increment at the plastic zone have been neglected in the model offered by Fahimifar and Zareinezhad (2009). However, Utunnel program has calculated the elastic strain increment at the plastic zone according to dilatancy angle. It has also taken into account the effect of variations in dilatancy angle on performance of the tunnel.

Elasto-plastic radius is increased by raising the dilatancy angle regarding the effect of this angle and elastic strain increment at the plastic zone. Meanwhile, according to keeping the pressure of lining constant within the proposed model, it seems that the ground deformation before installation of lining is considerably increased by raising the dilatancy angle.

The ratio between permeability at plastic zone to initial permeability K_r / K_o is noticeably high from the excavated wall to the elasto-plastic interface. It is changed from 1 at elasto-plastic interface to 16 at excavated wall in the model proposed by Brown and Bray. This ratio grows in *Utunnel* program by increasing the dilatancy angle. This ratio remains constant at 1 for dilatancy angle of 0°, while it is changed from 1 to 24 by increasing the dilatancy angle to 7.5°.

In Fig 3, the effect of different conditions of groundwater level on ground response curve, radial and tangential stresses have been demonstrated by keeping the dilatancy angle constant using Utunnel program.

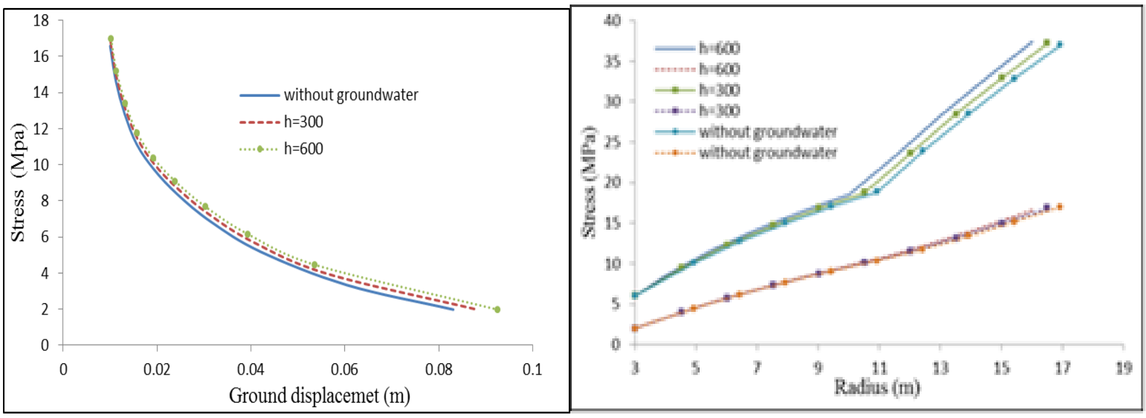

Fig 3: Ground response curves (right), Radial and tangential stresses at plastic zone (Plain lines denote tangential stress while dotted lines represent radial stress)(left), $\varphi = cte$

As evident from Fig. 3, small changes in the height of groundwater level do not significantly affect elasto-plastic radius and ground deformation. Due to the high initial stress of this tunnel and good quality of rock mass, limited variations in the groundwater level will not affect the behavior of surrounding rock mass considerably.

7. Conclusion

The following main conclusions can be drawn from this study:

In contrast with the method proposed by Brown and Bray, increments in elastic and plastic strains are calculated separately at each loop in this model. By increasing the dilatancy angle, the plastic strain will be intensified at each loop, while raising the plastic strain will add to deformation of the rock mass and also elasto-plastic radius. Taking into account the

separate calculation of elastic and plastic strains in the plastic zone, secondary permeability is correlated with square of the plastic strain. Therefore, the suggested model provides a more accurate criterion for considering hydraulic-mechanical interaction as compared to the model of Brown and Bray. Since both plastic and elastic strains are addressed in the plastic zone, the ratio of secondary permeability to initial permeability will be increased at greater dilatancy angles there.
Since application of the radial seepage model is inaccurate for shallow tunnels due to their significant error content, a combination of accurate non-radial model of Ming and Darcy's radial model is recommended to model distribution of pore pressure around the tunnel. A new model has been made for the distribution of pore pressure at elastic zone around the tunnel using modified model of Ming. Thereby, it would be possible to calculate the pore pressure at any point around the tunnel. Moreover, a model has been offered considering hydraulic-mechanical interaction and secondary permeability for the distribution of pore pressure at the plastic zone around the tunnel. Based on the results obtained from hydraulic analysis, both elasto-plastic radius and ground deformation are increased prior to installation of the lining by raising the groundwater level.

References

Alonso, E., Alejano, L. R., Varas, F., Fdez-Manin, G., & Carranza-Torres, C. (2003). Ground Response Curves for Rock Masses Exhibiting Strain-Softening Behavior. Internationa Journal of Numerical and Analytical Methods in Geomechanics, 27(13), 1153-1185.

Brown, E., & Bray, J. (1982). Rock-Support Interaction calculations for Pressure Shafts and Tunnels. ISRM Symposium, (pp. 26-28). Aachen.

Fahimifar, A., & Zareifard, M. R. (2009). A Thoretical Solution for Analysis of Tunnels below Groundwater Considering the Hydraulic-Mechanical Coupling. Tunnelling and Space Technology, 634-646.

Ghadami, H. (2012). Stability Analysis of Pressure Tunnels, Considering Seepage Effect. Master of Science Thesis, Tafresh University, Tafresh, Iran.

Hoek, E., & Brown, E. (1980). Empirical Strength Criterion for Rock Masses. Journal of Geotechnical Engineering., Asce 106 (GT9), 1013-1035.

Kolymbas, D., & Wagner, P. (2007). Groundwater Ingress to Tunnels - The Exact Analytical Solution. Tunnelling And Underground Space Technology, 23-27.

Ming, H., Meng-Shu, W., Tan, Z.-S., & Xiu-Ying, W. (2010). Analytical Solution for Steady Seepage into an Underwater Circular Tunnel. Tunnelling and Underground Space Technology, 391-396.

Park, K.-H., Tontavanich, B., & Lee, J.-G. (2008). A Simple Procedure for Ground Response Curve of Circular Tunnel in Elastic-Strain Softening Rock Masses. Tunnelling and Underground Space Technology, 151-159.

Timoshenko, S., & Goodier, J. (1994). Theory of Elasticity. New York: McGraw-Hill.

Wang, Y. (1996). Ground Response of Circular Tunnel in Poorly Consolidated Rock. Journal of Geotechnical Engineering, 122(9), 703-708.