A Study on the Effect of Flow Rule on the Bearing Capacity of Strip Foundations by Method of Stress Characteristics

Presenter:
Mehdi Veiskarami

Fatemeh Valikhah[1]andMehdi Veiskarami[2*]

[1]M.Sc. Student in Geotech. Eng., Department of Civil Engineering, University of Guilan, Iran
[2*]Assistant Professor, Department of Civil Engineering, University of Guilan, Iran
(Corresponding Author's Email: mveiskarami@gmail.com)

Abstract

Several methods can be employed to estimate the bearing capacity of shallow footings, e.g. the finite element method, the zero extension lines method, application of limit theorems, the method of stress characteristics and some different approaches. It is traditionally accepted that the bearing capacity of foundations are those obtained by an associated flow rule assumption. However, the angle of dilation, ψ, in the soils is usually much lower than the soil friction angle, ϕ. In fact, the non-associated flow rule governs the soil's behavior in reality. Such property of soils has been considered in this study and the method of stress characteristics is used to compute the bearing capacity of strip foundations. The flow rule affects the size of the plastic zone beneath the foundations and leads lower values for the ultimate capacity. In this research a fully smooth contact for the footing-soil interface has been assumed. Results based on the method of stress characteristics have been compared to those available in the literature.

Keywords: Bearing Capacity, Strip Foundations, Stress Characteristics, Plasticity, Flow Rule

1. Introduction

There are several different approaches in determination of the bearing capacity of shallow foundations. One of the first important contributions can be attributed to Prandtl (1920), who analyzed a rigid ideal plastic weightless half space subjected to a strip load, and Reissner (1924), who considered a similar problem which differed in two ways: implementation of a pure frictional materials and presence of a surcharge pressure at the ground surface. Terzaghi (1943) assumed a simplified approach for the state of failure when the soil weight is taken into account. He also conservatively suggested the global bearing capacity,q_{ult}, which has been used so far, as a sum of three independent terms, i.e. the term accounting for soil cohesion intercept, corresponding to N_c, the term accounting for the surcharge pressure, corresponding to N_q and the term accounting for the soil weight, corresponding to N_γ integrated as follows:

$$q_{ult} = c\,N_c + qN_q + 0.5B\gamma N_\gamma \tag{1}$$

In this equation, $c$is the soil cohesion intercept, q is the surcharge pressure, γ is the soil weight, B is the foundation width and $N_i$arebearing capacity factors. While the first two factors have been known to have exact theoretical solutions, the third one has a wide range of suggested values. As an example, Bowles (1996) made a literature study and found that the suggested values for the third bearing capacity factor, N_γ, varies between 38 and 192 for $\phi = 40°$.

Hill (1950) proposed new values of the third bearing capacity coefficient for smooth footing-soil interface and weightless soil with assuming different failure mechanisms. His mechanism was later on used by other researchers as a standard smooth base foundation collapse pattern. In 1951 and 1963, Meyerhof adopted a log-spiral failure mechanism and derived new values for the bearing capacity factors with some correction terms, i.e. the shape, depth and inclination factors. Sokolovskii (1960) employedthe method of stress characteristics for estimating the bearing capacity of foundations with considering the weight of soils corresponding to the vertical body force, γ.Hansen (1970) extended Meyerhof's equations and proposed the equations of the bearing capacity for both shallow and deep foundations. Vesić (1973) and Chen (1975) also extended the equation of bearing capacity with expressing the new equation forN_γ. Bolton and Lau (1993) performed a rigorous study on the influence of soil weight and the boundary conditions on the bearing capacity factors and suggested all bearing capacity terms for both circular and strip foundations with smooth and rough bases. Comparisons show that their suggested values are rather the uppermost values to previous ones. Michalovski (1997) applied the kinematic approach of the limit analysis and computed the bearing capacity factors for both associated and non-associated flow rules. It was one of the first attempts in implementation of the flow rule in determination of the bearing capacity. There are a number of more recent attempts for special cases and different boundary conditions (e.g., Kumar and Ghosh, 2005; Kumar and Kouzer, 2007; Kumar, 2009 and Veiskarami *et al*., 2011).

2. Theory

The method of stress characteristics or the slip lines method which was founded by Sokolovskii in 1960s, is a famous method for solving plasticity problems in soil. The bearing capacity of foundations can be obtained by this method where both equilibrium and yield equations are satisfied simultaneously. Sokolovskii founded the important fact that the characteristics directions of the related hyperbolic type partial differential equations of equilibrium in terms of σ and θ (defined first by Kötter, 1903), when combined by the yield criterion, coincided with the direction of shear planes. Therefore, the method was given the name of the "method of stress characteristics". Although a few decades later, similar directions were found by Habibagahi and Ghahramani (1979) and Jahanandish *et al.* (1989) for strain characteristics to find both stresses and strains through one computational attempt. Mohr stress circle is useful to decreasing the variants of primer equations and forming the characteristics equations and characteristics directions. With solving the characteristics equations in partial differential system, four unknowns, x, z, σ, and θwill be found. The finite difference method can be used to solve these equations simultaneously.

The stress characteristics equations can be written as follows (based on Sokolovskii's equations in 1960 and according to representation of Anvar and Ghahramani in 1997):

Along the positive direction (σ^+):

$$\begin{cases} \frac{dz}{dx} = \tan(\theta + \mu) \\ d\sigma + 2(\sigma tan\phi + c)d\theta = -X(tan\phi dz - dx) + Z(tan\phi dx + dz \end{cases} \quad (2)$$

Along the negative direction (σ^-):

$$\begin{cases} \frac{dz}{dx} = \tan(\theta - \mu) \\ d\sigma - 2(\sigma tan\phi + c)d\theta = +X(tan\phi dz + dx) - Z(tan\phi dx - dz \end{cases} \quad (3)$$

Where: $\mu = \frac{\pi}{4} - \frac{\phi}{2}$

σ is the mean effective stress, $\theta$is the angle defining the direction of the major principal stress direction (orminor principal stress plane) with horizontal direction, c and $\phi$are soil shear strength parameters, $X$and Zarecomponents of the horizontal and vertical body forces correspondingly, $x$and $z$are coordinates of each point respectively. Figure 1, shows the state of stress and directions of the stress characteristics on a Mohr circle as well as an element of soil in the ground.

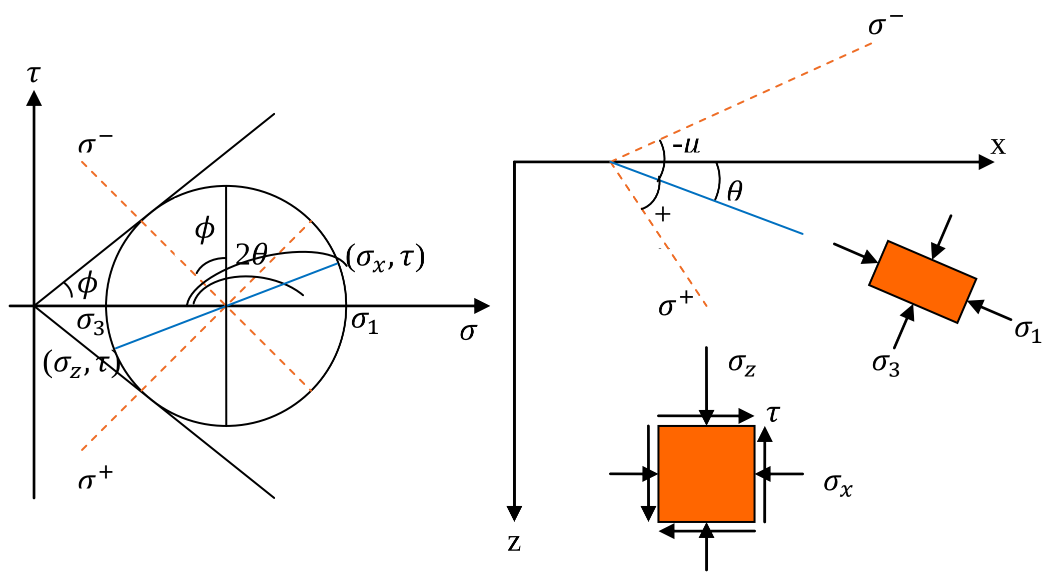


Figure 1Thestate of stress on Mohr circle and the soil body along the directions of the stress characteristics

Harr (1966) indicated that there are three zones beneath the foundation; the passive (or Rankine) zone, the transition (or Goursat) zone and the mixed-boundary (or Coulomb) zone that they have been identified corresponding boundary conditions. These different zones can be seen in Figure 2.This problem can be solved with the finite difference method and using three-point strategy, i.e. having known all information on two points, the unknowns will be found in the third point obtained by intersecting two plus and minus characteristics lines emerging from previous points.

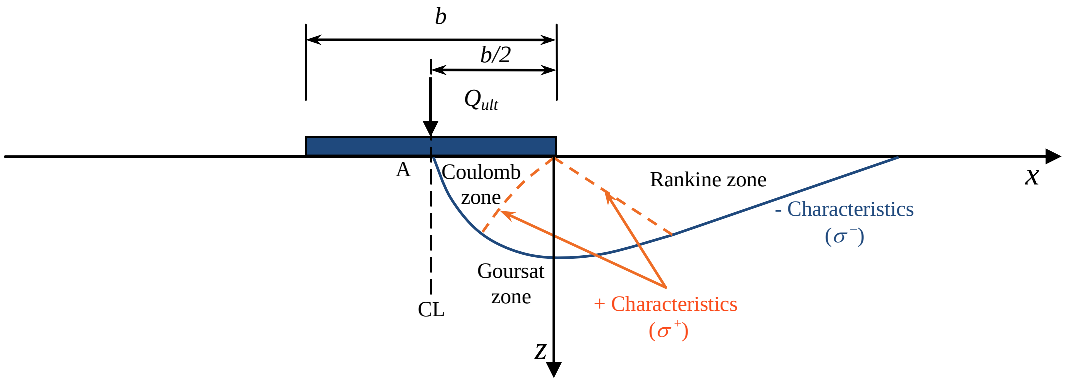


Figure 2 Stress characteristics and different zones formed beneath the foundation

Method of stress characteristics can be applied to find the variation of N_γ with footing roughness (Kumar, 2009), bearing capacity of unsaturated soils (Veiskrami *et al.*, 2009 and Jahanandish *et al.*, 2010), finding end-bearing capacity of driven piles in sand (Veiskarami, *et al*, 2011) and recently, estimation of the bearing capacity of foundations subjected to groundwater flow (Veiskarami and Kumar, 2012).

A computer code is then developed in to employ the stress characteristics equations. Prior to further analyses, the convergence of the results was checked to find the optimum number of points which gives rise to a reasonable approximation of the solution, i.e. with less than 1 to 2% error. Figure 3 shows the convergence of the numerical solution against the number of points which indicates a rather rapid convergence. It was found that if the total number of divisions in the Rankine zone exceeds say, 50 points. For the rest of analyzed cases, the total number of divisions in Rankine zone (and others) was taken as 60 points which seems to be quite proper.

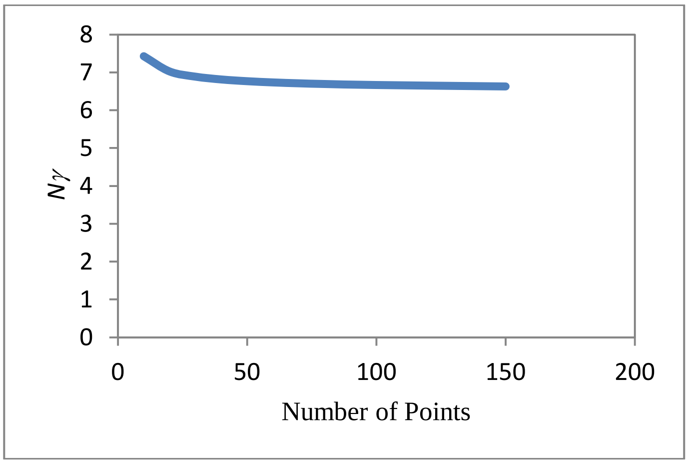


Figure 3Convergence of the value of Nγ with increase the number of points in MATLAB for ϕ = 30° and ψ = 15°

The influence of flow rule is very important in estimation of the limit load based on plasticity equations. An associated flow rule, as in many methods, is the basis of the stress characteristics method. It is conventional to find the solution based on an associated flow rule assumption but in fact, a non-associated flow rule governs the actual behavior of most of soils. This effect is important since the non-associated flow rule requires less volume change and hence, less extension of the plastic zone beneath the foundation resulting in

lower bearing capacity (or earth pressure on retaining structures). Therefore, the non-associativity should be taken into account if a more proper estimation of the bearing capacity is required.
To include the influence of flow rule, one significant step was taken by Drescher and Detournay (1993) in computing equivalent values for soil cohesion and friction angle in case of a non-associated flow rule. Equivalent values can be computed by the following equations derived independently by Davis (1968) and Rowe (1969) based on equating the rate of plastic work for a non-associative and an equivalently associative material:

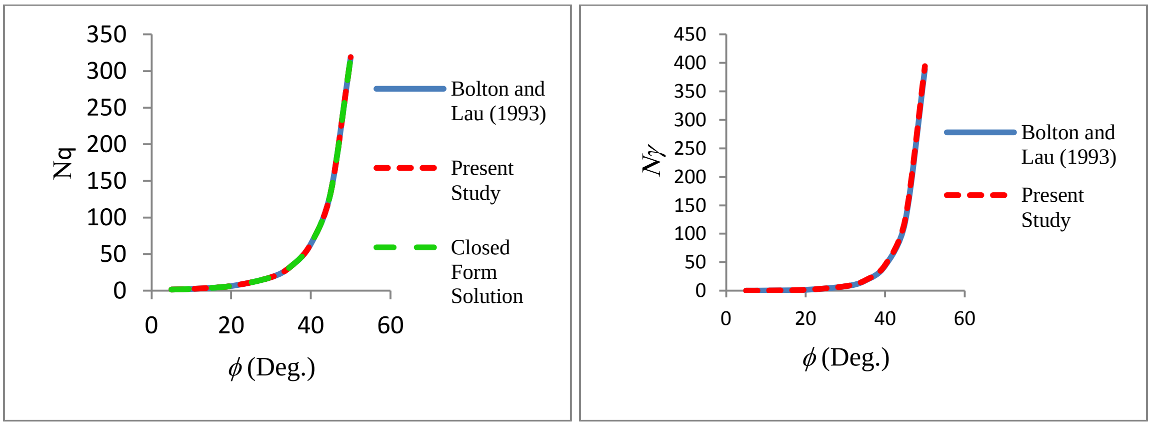
$$tan\phi^* = \frac{cos\psi sin\phi}{1-sin\psi sin\phi} \qquad (4)$$

$$c^* = c\frac{cos\psi cos\phi}{1-sin\psi sin\phi} \qquad (5)$$

In this equationψ is dilation angle, ϕ is actual friction angle, c is cohesion, c^* is the modified (or equivalent) cohesion due to non-associativeness and ϕ^* is the modified (or equivalent) friction angle for spotting the non-associativity of soils.

3. Results

In this study, the non-associativity effect on the bearing capacity of smooth base strip foundations on sand (cohesionless soils) has been investigated and reduced values of the bearing capacity factors have been computed for different soil angle of dilations. Method of stress characteristics along with the assumptions of Drescher and Detournay (1993) is used to find the bearing capacity factors. A computer code in MATLAB was developed to solve the system of partial differential equations by a finite difference technique and results were presented in charts and graphs in comparison to those available in the literature. First, the results were obtained for associative soils possessing different friction angles and compared to those results reported by closed-form solution and also by Bolton and Lau (1993) with the same method and similar assumptions. Figure 4 shows this comparison and it is clear that the results comply well with available verified data and hence, the code works properly.



(a) (b)

Figure 4Comparison of a) $N_q$and b) N_γ with closed-form solutions and Bolton and Lau's (1993) results for smooth bases

In the next step, the results for both associated and non-associated cases were computed. In Figure 5 the effect of non-associativity on the plastic zone beneath the foundation is shown which clearly indicates a smaller plastic zone with lower angle of dilation (or higher non-associativity). In this figure the width of the foundation is kept constant as 1.0m. These results are for spatial fixed soil friction angle but different dilation angles. It is apparent that as the dilation angle increases from 0 to the maximum value (equal to the soil friction angle) the plastic zone increases in size. As a result, the passive earth pressure resisting the failure of the foundation increases and analogously, the bearing capacity itself.

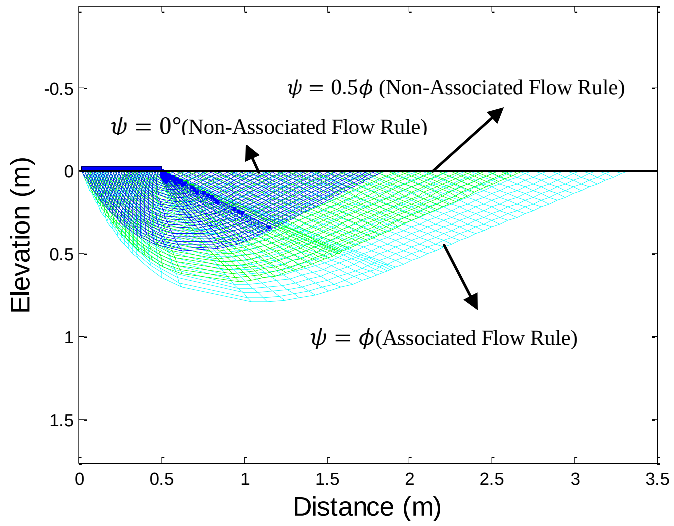
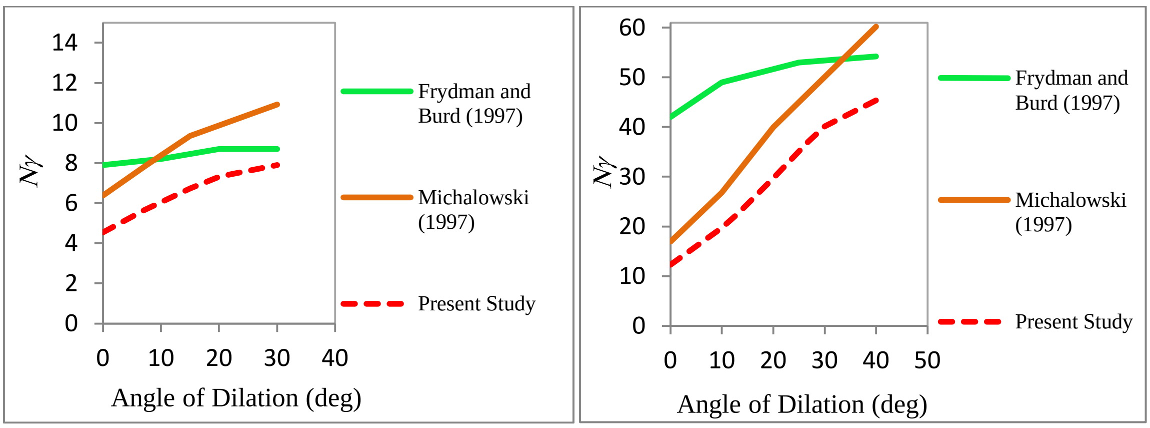


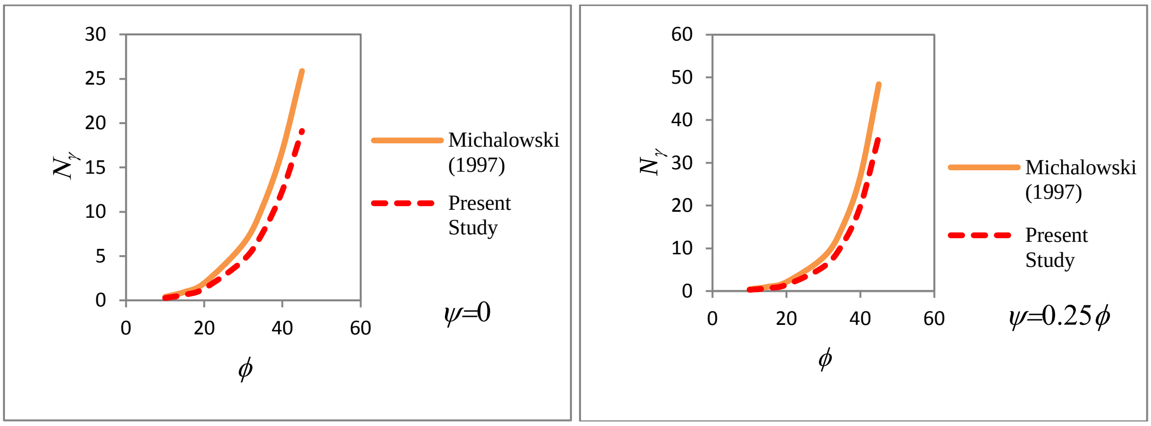
Figure 5 Snapshots of the analyzed model for $\phi = 45°$ for both associated and non-associated flow rule

It is remarkable that this effect is more significant in rough base foundations. The values of the third bearing capacity factor were computed for two constant friction angle, $\phi = 30°$,$\phi = 40°$ and a variety of the dilation angles to investigate the effect of non-associativity on the bearing capacity of foundations. These results are presented in Figure 6.For the comparison purpose, the values obtained by Frydman and Burd (1997) by the finite element method and those computed by Michalowski (1997) by the kinematic approach of the limit analysis are presented in the same table. Although all values are more or less close to each other, the stress characteristics method gives the lowermost of all values as it was expected, since its assumptions are much closer to the lower-bound limit theorem requirements.In Figure 7 the values of the bearing capacity factor,N_γ, have been presented for different friction angles and different angles of dilation in a comparative manner. It is evident that the method of stress characteristics gives rather more conservative results than other methods based on the requirements of the upper-bound theorem.

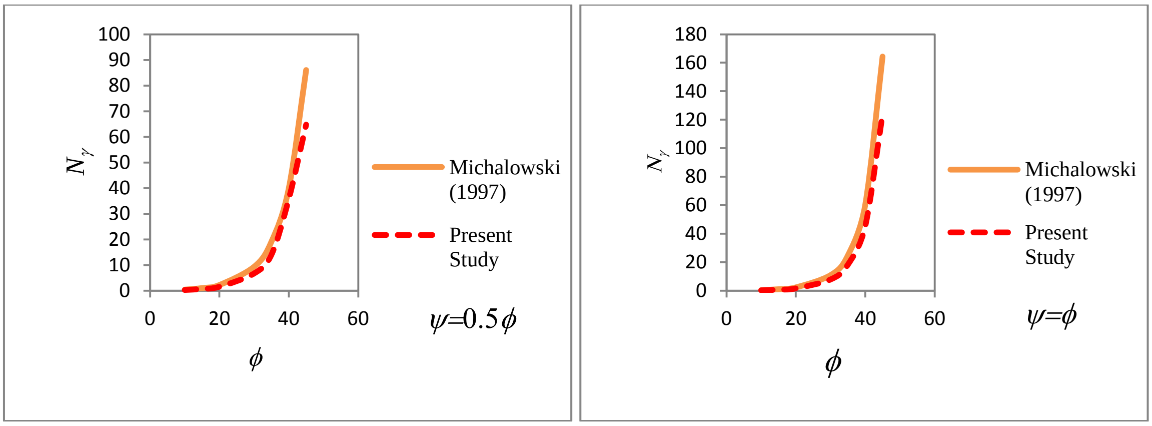


(a) (b)

Figure 6Values of N_γ for smooth interface for a)$\phi = 30°$ and b)$\phi = 40°$ for different dilation angles and comparison with the results of Michalowski (1997) and Frydman and Burd (1997)



(a) (b)



(c) (d)

Figure 7 Comparison of the results for N_γ considering the non-associative flow rule with those presented by Michalowski (1997) for smooth bases: a) $\psi = 0°$, b) $\psi = 0.25\ \phi$, c)$\psi = 0.5\phi$ and d) $\psi = \phi$

Finally, the method to compute the bearing capacityof surface footings based on the non-associativity effect considerations, was applied to estimate the bearing capacity of some case studies found in the literature. Footing load test data of Cerato (2005) was compiled for this study. Table 1 presents properties of two types of sand used in her studies along with the size and properties of the foundation in the footing load test program carried out in University of Massachusetts, Amherest, MA, USA in 2004-2005.

No.	Type of Sand	Shape	B (m)	Dr (%)	γ(kN/m^3)
1	Brown Mortar Sand	Circular	0.1016	70	16.2
2	Brown Mortar Sand	Circular	0.1016	43	15.2
3	Winter Sand	Circular	0.1016	87	19.2
4	Winter Sand	Circular	0.1016	57	18.3

Table 1Sand properties and foundation size (Cerato, 2005)

It is now required to define the relationship between the soil angle of dilation, governing the non-associative nature of the soil and the stress level. Since this relationship is almost complex to be defined regarding complex stress distribution beneath the foundation, the approach suggested by Meyerhof (1950) has been considered. He assumed that the mean value of the normal stress, $\sigma_{g,m}$,along the slip lines is equal to about 1/10 of the ultimate capacity. His procedure is illustrated in Figure 8.

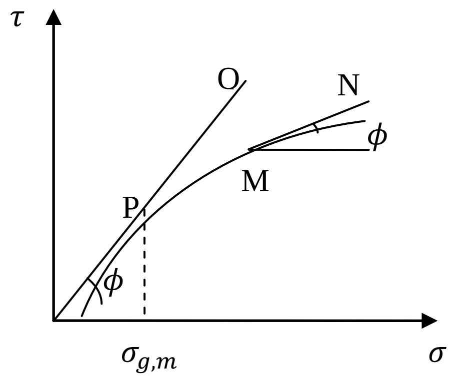


Figure 8Meyerhof's procedure for calculating the maximum mobilized friction angle controlling the bearing capacity of a loaded foundation

Now, the angle of dilation can be found based on the empirical equation of Bolton (1986) for sands as a function of soil density index, D_r and stress level:

$$\phi_{max} = \phi_{c.s.} + 0.8\psi_{max}$$

$\phi_{max} = \phi_{c.s.} + 5I_R$ (in plane strain condition)

$\phi_{max} = \phi_{c.s.} + 3I_R$ (in triaxial condition)

$$I_R = D_r(Q - \ln(\sigma)) - R \quad (6)$$

In these equations, ϕ_{max} is the maximum mobilized friction angle, $\phi_{c.s.}$ is the critical state friction angle, ψ_{max} is the maximum dilation angle, I_R is the dilatancy index, D_r is soil relative density (in decimals), σ is the effective stress (in KPa), Q and R are constant. With Bolton's recommendation in 1986 in this study Q=10 and R=1 are assumed.

In Table 2 computed and measured values of the bearing capacity factor, N_γ, are presented and computed. Since the test condition in finding the soil friction angle was not available, assumptions for both plane strain and triaxial tests were made and analyses were carried out for both assumptions. It can be observed that results compare reasonably with those obtained experimentally. Moreover, the results are in no case higher than those observed in the laboratory. Therefore, the non-associativity assumption is quite conservative. Moreover, computations based on an associated flow rule assumption indicate an unsafe estimate of the bearing capacity and hence, assuming an associated flow rule in computation of the bearing capacity should be made with care.

No.	ϕ_{peak} (deg)	N_γ	ψ(deg)		N_γ(Present Study -non-associated flow rule)		N_γ(Present Study-associated flow rule)
			Plane Strain condition	Triaxial condition	Plane Strain condition	Triaxial condition	
1	38	728	23.977	14.385	701.6317	701.0229	789.8510
2	34	593	13.0108	7.8064	581.2797	544.2728	685.4208
3	46	682	30.7322	18.4395	659.2556	618.8677	709.7392
4	41.5	532	19.0781	11.445	494.2228	527.2200	542.9384

Table 2Comparison of results with case studies

4. Conclusions

Observations confirmed that most of soils obey a non-associated flow rule and hence, further considerations should be taken to account for this inherent property of the soil. In particular, in stability problems in soil, an associated flow rule assumption may give rise to unsafe estimation of the limiting load. It is evident that the non-associativity effect reduces the size of the plastic zone resulting in a less capacity in comparison to that corresponding to an associated flow rule material. This effect is very important when the bearing capacity of foundations is to be estimated and ignoring the non-associativity may give rise to over-estimation of the bearing capacity.

The non-associativity of the soil was considered by an equivalent yield surface accounting for the effect of angle of dilation. The method of stress characteristics was used to implement this effect. A computer code in MATLAB was developed to solve the equations of the method of stress characteristics by a numerical finite different scheme and compared with some available numerical results, showing good consistency. Thereafter, the procedure was applied to some available case studies of footing load tests on sand indicating the importance of the non-associativity effects. When this effect is not considered, an unsafe (over-estimated) bearing capacity was obtained whereas the non-associated flow rule showed always conservative and reasonable results in comparison to experimental data.

5. References

Anvar, S.A., and Ghahramani, A., 1997, Equilibrium equations on zero extension lines and their application to soil engineering, *Iranian Journal of Science and Technology*, 21(1): 11–34.

Bolton, M.D. 1986. The strength and dilatancy of sands. *Géotechnique*, 36(1): 65–78.

Bolton, M.D., and Lau, C.K., 1993, Vertical bearing capacity factors for circular and strip footings on Mohr Coulomb soil. *Can. Geotech. J.*, 30 (6): 1024-1033.

Bowles, J.E., 1996. Foundations analysis and design, 5th ed. *McGraw- Hill*, New York.

Cerato, A.B. and Lutenegger, A.J.,2007, Scale effects of shallow foundation bearing capacity on granular material, *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 133(10), pp. 1192–1202.

Clark, J.I. 1998. The settlement and bearing capacity of very large foundations on strong soils: 1996 R.M. Hardy Keynote address, *Canadian Geotechnical Journal*, 35, pp. 131–145.

Davis, E. H., 1968, Theories of plasticity and the failure of soil masses, in: Soil Mechanics Selected Topics, Edited by I. K. Lee, *Butterworth*.

Drescher, A., and Detournay, E., 1993. Limit load in translational failure mechanisms for associative and non-associative materials. *Géotechnique*, 43(3): 443–456.

Frydman, S., and Burd, H.J. 1997. Numerical studies of the bearing capacity factor, Ng, *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 123(1): 20–29.

Hansen, J.B. 1970. A revised and extended formula for bearing capacity, *Danish Geotechnical Institute, Copenhagen, Bulletin*, 28, pp. 5–11.

Harr, M.E., 1966. Foundations of theoretical soil mechanics. *McGraw-Hill, New York.*

Jahanandish, M., Veiskarami, M., and Ghahramani, A. 2010. Effect of stress level on the bearing capacity factor, $N\gamma$, by the ZEL method, *KSCE Journal of Civil Engineering*, 14(5):709-723.

Kimura, T., Kusakabe, O. and Saitoh, K. 1985. Geotechnical model tests of bearing capacity problems in a centrifuge, *Géotechnique*, 35(1), pp. 33–45.

Kumar, J. 2009. The variation of Ng with footing roughness using the method of characteristics, *International Journal for Numerical and Analytical Methods in Geomechanics*, 33(2): 275–284.

Kumar, J., and Ghosh, P., 2005, Bearing capacity factor Nγ for ring footings using the method of characteristics, *Can. Geotech. J.*, 42: 1474–1484.

Kumar, J., and Kouzer, K. M.,2007, Effect of footing roughness on bearing capacity factor $N\gamma$,*J. Geotech. Geoenviron. Eng.*, ASCE, 133(5), 502-511.

Meyerhof, G.G. 1950. The bearing capacity of sand. Ph.D. thesis, University of London, London.

Michalowski, R.L. 1997. An estimate of the influence of the soil weight on the bearing capacity using limit analysis, *Soils and Foundations*, 37(4): 57–64.

Rowe, P. W., 1969, The relation between the shear strength of sands in triaxial compression, plane strain and direct shear, *Géotechnique*, 19(1): 75-86.

Sokolovskii, V.V., 1960 Statics of Soil Media, *Butterworth*, London, Translated from the 1942 Russian edition.

Veiskarami, M. 2010 Stress level based prediction of load displacement behaviour and bearing capacity of foundations by ZEL method, Ph.D. Dissertation Submitted to the Department of Civil Engineering, Shiraz University, Shiraz, Iran.

Veiskarami, M., Jahanandish, M., and Ghahramani, A. 2011. Prediction of the bearing capacity and load-displacement behaviour of shallow foundations by the stress-level-based ZEL method, *Scientia Iranica*, Elsevier, 18(1): 16–27.

Veiskarami, M., Jahanandish, M., and Ghahramani, A. 2011. Stress level based bearing capacity of foundations: Verification of results with 131 case studies, *KSCE Journal of Civil Engineering*, 16(5):723-732.

Veiskarami, M., Kumar, J.2012. Bearing capacity of foundations subjected to groundwater flow, *Geomechanics and Geoengineering*: *An International Journal*, Taylor and Francis, pp. 1-9. doi:10.1080/17486025.2011.631038.

Veiskarami, M.,Eslami, A., Kumar, J. 2011. End-bearing capacity of driven piles in sand using the stress characteristics method: analysis and implementation, *Can. Geotech. J.* 48 (10):1570-1586.