
# Stability Assessment of Concrete Gravity Dams Using Fracture Mechanics Criteria; Case Study Zavin Dam

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## ABSTRACT

All concrete dams suffer from cracking which is caused by various factors such as construction, poor curing, volume changing in mass concrete and load application. These cracks may propagate and leading to significant change in the failure resistance of dam.

Recently, fracture mechanics has been advocated in papers, as an alternative means to assess dam safety. In this study, linear elastic fracture mechanics concepts were applied to assess the crack stability in ZAVIN Concrete Gravity Dam that is subjected to five various loading conditions in full and empty reservoir.

The two dimensional maximum section of ZAVIN Dam was modeled using FRANC2D software, which is specially prepared for fracture mechanics analysis. A finite element linear elastic fracture mechanics analysis in plain strain condition was used to determine the stress intensity factor at different crack positions and maximum circumferential stress to compute crack stability and kinking angle of cracks at each step of crack propagations in the dam body. The results of this study would be useful to prevent possible instability of ZAVIN and other concrete gravity dams.

**Keywords:** Concrete Dam, Crack Propagation, Fracture Mechanics, Stress Intensity Factor.

## INTRODUCTION

Cracks are among the most common flaws in many practical civil engineering structures. In Concrete dams the cracks are caused by various factors such as volumetric change, load application, inadequate design and construction. Therefore, assuming that no existing dam is completely without cracks, the question of whether the safety of such dam is guaranteed appears to justify.

Most of the existing dams were designed on the basis of the classical beam analysis ( $\sigma$ =P/A $\pm$ MC/I); such an equation only valid for “shallow beams” cannot be blindly applied to a dam without certain shear correction, and it does not recognize the singular nature of the stress at the crack tip [1-5].

In response to the challenge of evaluating older dams, recently fracture mechanics has been strongly advocated in journals as an alternative means to assess dam safety and crack stability in dams [1-6]. Further more, this approach has also been recognized as a valid approach and one that should be investigated prior to major rehabilitation of cracked massive concrete structures [1-3].

The phenomenon of decreased structural fracture strength with increased size of brittle structures is schematically represented in Figure 1. According to strength criteria used in elastic, plastic or elastoplastic design $\sigma_N$ is independent of the structural size, and hence, the strength of materials (SOM) approach cannot correctly predict the load-bearing capacity of brittle structures. Fracture mechanics approaches can capture the size effect in the ultimate strength of brittle structures. For linear elastic fracture mechanics (LEFM), $\sigma_N$ varies inversely with $\sqrt{d}$ and the plot of Log $\sigma_N$ versus Log d is a straight line of slope (-1/2).

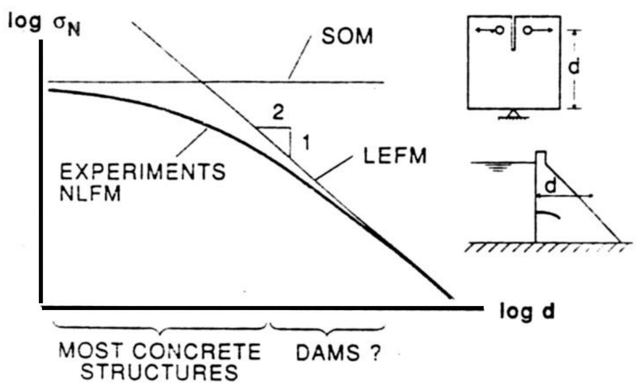


**Fig. 1-** structural size effect [4]

However LEFM overestimates the load-bearing capacity of concrete structures of normal dimensions. Within the assumption of LEFM, at the tip of crack propagation in pure opening mode (mode I) in a brittle material, a stress singularity arises which is characterized by the stress intensity factor (SIF).The stress components for the infinite cracked plate under tension using westergaard complex stress function are [4]:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} Cos\frac{\theta}{2}\left(1 - Sin\frac{\theta}{2} Cos\frac{3\theta}{2}\right) \tag{1}$$
$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} Cos\frac{\theta}{2}\left(1 + Sin\frac{\theta}{2} Cos\frac{3\theta}{2}\right)$$
$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} Cos\frac{\theta}{2} Sin\frac{\theta}{2} Cos\frac{3\theta}{2}$$

Where KI is the stress intensity factor (SIF) for tension mode may be defined as:

$$K_I = Lim_{r\to 0}\sqrt{2\pi r}.\sigma_y(r, \theta = 0) \tag{2}$$

For the infinite cracked plate this is:

$$K_I = \sigma\sqrt{\pi . a} \tag{3}$$

The displacements at the crack tip for the plane stress condition are similarly obtained as:

$$u = 2(1+\upsilon)\frac{K_I}{E}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}(1 - 2\upsilon + \sin^2\frac{\theta}{2}) \tag{4}$$
$$v = 2(1+\upsilon)\frac{K_I}{E}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}(2 - 2\upsilon - \cos^2\frac{\theta}{2})$$

So under linear elastic fracture mechanics (LEFM) assumption, the stress, strain and displacement fields in the near crack-tip region are determined by the stress intensity factors. Therefore fundamental to the use of the finite element method for LEFM is the extraction of accurate SIF's from the finite element results.

Techniques for extracting SIF's fall into two directories; direct approaches which correlate SIF's with finite element results directly, and energy approaches which first compute energy release rates. In general the energy approaches are more accurate and should be used preferentially. However, the direct methods have utility and the expressions are simpler. Some methods for extracting SIF's are:

**a) Displacement Correlation Method (DCT)**

This is one of the simplest and historically on of the first techniques used to extract SIF's from FE results (Chan, 1970) and is a direct approach. The configuration for this approach is shown in figure 2 and the expressions for the SIF's using plane strain assumptions and quarter-point elements (QPE) are:

$$K_I = \frac{\mu.\sqrt{2\pi}}{\sqrt{r}(2-2\upsilon)}.\left[4(v_b - v_d) + v_e - v_c\right] \tag{5}$$

$$K_{II} = \frac{\mu.\sqrt{2\pi}}{\sqrt{r}(2-2\upsilon)}.\left[4(u_b - u_d) + u_e - u_c\right]$$

Where $\mu$ is the shear modules, $\upsilon$ is Poisson's ratio, r is the distance from the crack tip to the correlation point, and $u_i$, $v_i$ are the X, Y displacements at point i. The same expression can be used for plane stress assumption if $\upsilon$ is replaced with $\upsilon = \upsilon/(1+\upsilon)$.

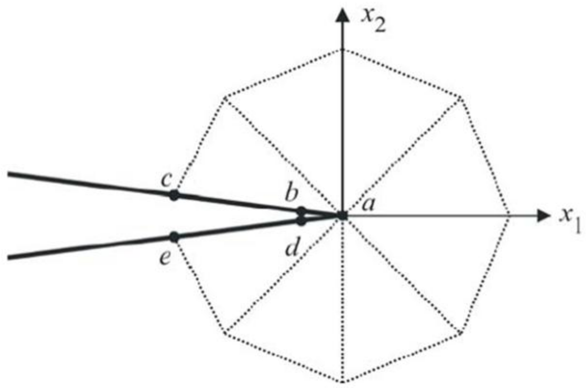


**Fig. 2**-QPE's with DCT approach

**b) Virtual Crack Extension Method**

This method is an energy approach that computes the rate of change in the total potential energy of a system for a small (virtual) extension of the crack under LEFM assumptions; this is equal to the energy release rate. This method was first proposed by parks (1975) and Helen (1975). The energy release rate for a small crack extension with constant external forces during crack growth is:

$$G = \frac{\partial \Pi}{\partial a} = \frac{1}{2}.U^T.\frac{\partial K}{\partial a}U \tag{6}$$

Where $\Pi$ is the total potential energy of a finite element system (with no body forces), U is the nodal displacement vector and K is the stiffness matrix. SIF's can then be computed from the simple relations:

$$K_I = \sqrt{G_I.E} \qquad K_{II} = \sqrt{G_{II}.E}, \qquad \text{For plane stress} \tag{7}$$

$$K_I = \sqrt{G_I.E/(1-\upsilon^2)} \qquad K_{II} = \sqrt{G_{II}.E/(1-\upsilon^2)}, \qquad \text{For plane strain}$$

**Modified Crack Closure Integral (MCCI)**

The MCCI technique was originally proposed by Rybki and Kanninen (1977). The resulting expressions for the energy release rate for the QPE's in this method are (see figure 3):

$$G_I = \frac{1}{\Delta L}\left[(C_{11}F^a{}_y + C_{12}F^f{}_y + C_{13}F^g{}_y)(u^b{}_y - u^e{}_y) + (C_{12}F^a{}_y + C_{22}F^f{}_y + C_{23}F^g{}_y)(u^c{}_y - u^d{}_y)\right] \tag{8}$$

$$G_I = \frac{1}{\Delta L}\left[(C_{11}F^a{}_y + C_{12}F^f{}_y + C_{13}F^g{}_y)(u^b{}_y - u^e{}_y) + (C_{12}F^a{}_y + C_{22}F^f{}_y + C_{23}F^g{}_y)(u^c{}_y - u^d{}_y)\right]$$

Where,

$$C_{11} = \frac{33\pi}{2} - 52, C_{12} = 17 - \frac{21\pi}{4}, C_{13} = \frac{21\pi}{2} - 32$$

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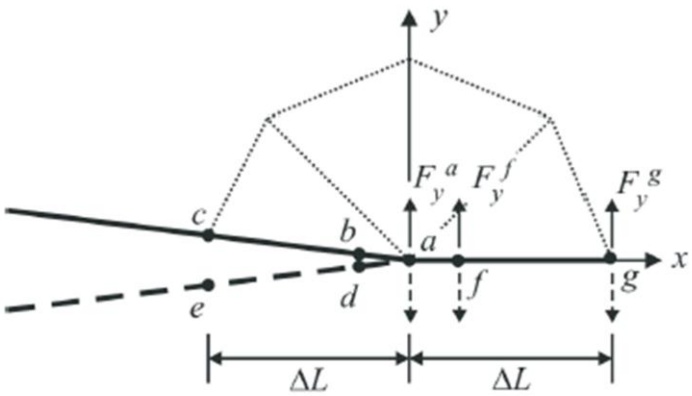



**Fig. 3-**QPE's with MCCI approach

**c) The J-Integral Method**

This is a well known nonlinear fracture mechanics parameter (Cherepanov, 1967; Rice, 1968; Budiansky, 1973). Under LEFM assumption, the J-Integral can be interpreted as being equivalent to the energy release rate. Using a crack coordinate system where the $X_1$ axis is tangential to the crack and the $X_2$ axis is perpendicular to the crack; J-Integral is defined as:

$$J = Lim_{\Gamma \to 0} \int_{\Gamma} \left[ W.n_1 - \sigma_{ij} . \frac{\partial u_i}{\partial x_1} .n_j \right] .d\Gamma \quad (9)$$

Where W is the strain energy density, $\sigma$ is the stress tensor, n is the unit outward normal to the contour, and u is the displacement vector (see figure 3). The contour integral in this simple form can be shown to the path independent. Providing there are no body forces inside the integration area, there are no tractions on the crack surface and the material behavior is elastic.

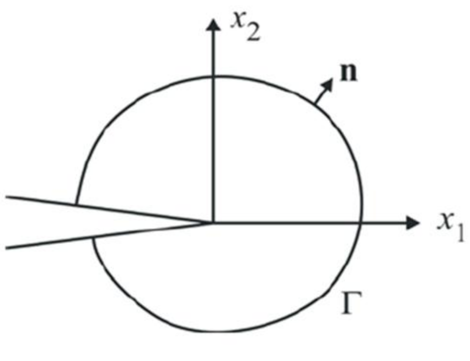



**Fig. 3-**J-Integral approach

The J-integral approach is the most accurate and should be used preferentially. It is aided if one has access to the FE programs for shape functions and numerical integration. Often acceptably accurate results can be obtained using the MCCI approach that uses only nodal displacements and forces, which are standard outputs from most FE programs. The DCT technique is the least accurate but is the simplest method.

According to LEFM criterion, crack propagation occurs as soon as the KI reaches its critical value KIc, which is assumed to be material property. Unstable crack propagation (usually leading to ultimate structural failure) occurs when KI=KIc for the critical value of the crack length, say ac. This condition, in fact, implies that any increase of the crack length due to fracture propagation would increase the SIF beyond its value.
The ratio of structural dimension d, to characteristic length $L_{ch}$ was found to be the dominant parameter for describing size effects using the fictitious crack model. LEFM is considered to be applicable for large values of ($d/L_{ch}$). Analysis of tree-point concrete bending beams showed that LEFM may not be valid for $d/L_{ch} < 25$ [4]. A ratio of $d/L_{ch}$ =25 results in a 5% difference between the load-caring capacities predicted by LEFM and fictitious crack model [4].
Facture Mechanics Concepts have been successfully applied to study the cracking phenomena in dams [1-6]. However, almost all the investigations have been made to assess the crack stability at the interface of concrete /rock foundation [1-5,7] and very few studies is reported regarding the cracks at the other places such as crest of dam, corners, around the inspection galleries etc [6].
In this study six different initial cracks were placed at the region of tensile stresses during five different load cases in an existing concrete gravity dam. Then, an incremented analysis based on Linear Elastic Fracture Mechanics was performed to determine crack stability and crack trajectory. The recently developed software "FRANC2D" was used to calculate stress Intensity factor and maximum circumferential stress to compute crack stability and kinking angle of cracks. The results of this study would be useful to assess dam safety and prevent instability of concrete gravity dams.

## DAM DESCRIPTION

The ZAVIN Concrete Gravity Dam is located in the northeast of Iran and was completed in 2002 (Figure 4). The main geometric data of the ZAVIN Dam listed in Table 1 [9].

**Tab. 1-** Geometry of ZAVIN Dam. [9]

| | |
|---|---|
| Dam height | 51.2 m |
| Crest length | 142 m |
| Crest thickness | 5 m |
| Base thickness | 53 m |

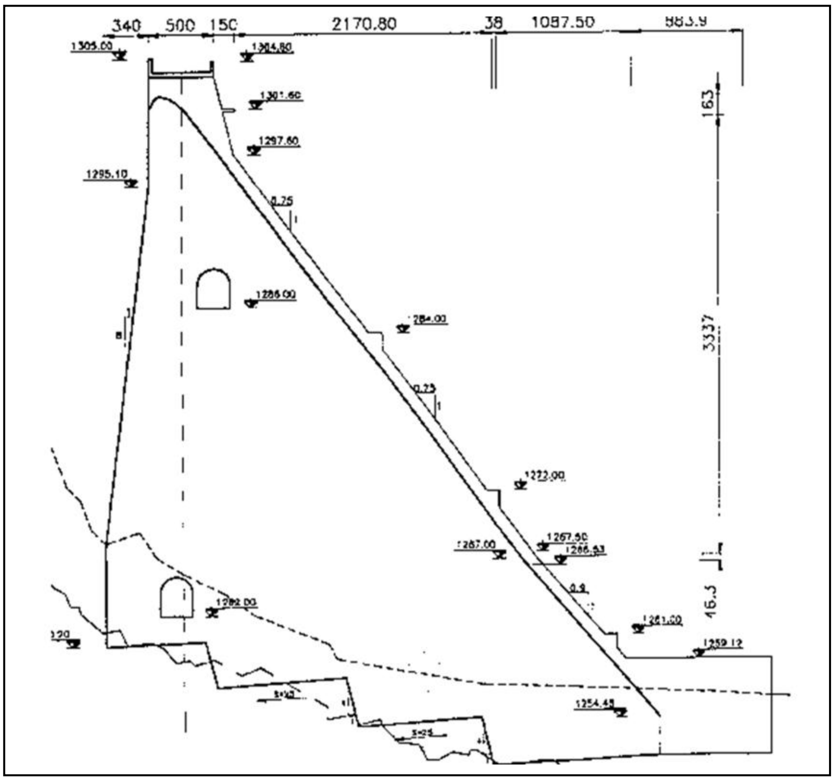



**Fig. 4-** Central Cross Section of ZAVIN Dam [9]

Two different kinds of finite element models were developed. One coarse discretized model and a model with finer discretization in the region, where stress concentration will occur. The finite element mesh for the dam and a sufficient portion of the foundation developed on the basis of quadratic Isoparametric 8 node and Triangular 6 Node elements as Table 2. A fully fixed boundary condition was assumed around the perimeter of F.E. model (see Figure 5).

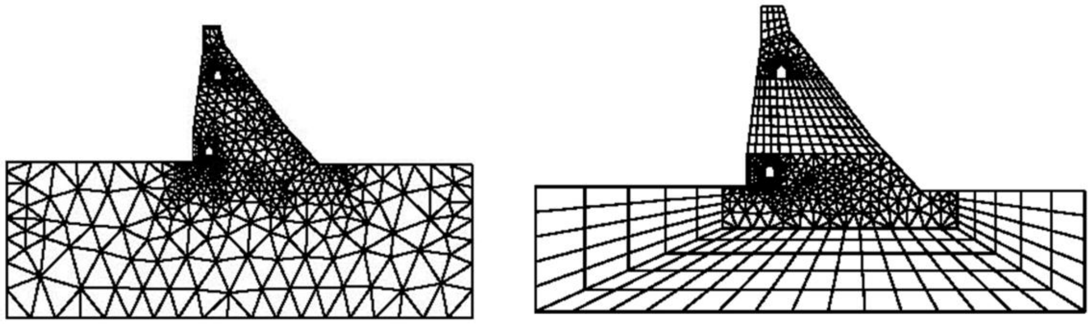


**Fig. 5**-F.E. Meshes for ZAVIN Dam

**Tab. 2-** FE Mesh discretization

| F.E. Mesh | Nodes | Elements |
|---|---|---|
| Coarse Mesh | 2100 | 935 |
| Fine Mesh | 2215 | 1042 |

## MATERIAL PROPERTIES

The material properties adopted in the analysis are shown in Table 3. The values of parameters are obtained from the material tests at the dam site [9], except fracture toughness. Since there is no experimental data available for the fracture toughness of ZAVIN Dam, three estimated values assumed based on the other data in the literature [2,3,10,11].

**Tab. 3-** Material properties

| Material properties | Elastic Modulus (MPa) | Poisson ratio | Density $(kg/m^3)$ | Fracture toughness (MPa $\sqrt{m}$ ) |
|---|---|---|---|---|
| Concrete | 2100 [9] | 0.17 [9] | 2400 [9] | 1.05 [3, 6]<br>2.3 [2] |
| Rock | 4200 [9] | 0.2 [9] | - | 0.81 [10,11] |
| Interface | 2100 [5] | - | - | 0.3 [11]<br>0 [3]<br>2.3 [2] |

For the rock foundation and dam concrete, an isotropic and homogeneous behavior was assumed. The rock was assumed as massless during analysis steps and contact elements used in the interface between dam and foundation.

## LOAD APPLICATION

For this model five different loading situations were analyzed to assess the dam safety. Load combinations were considered according to the USBR (1976)[1] and MOPU (1967)[2] as Table 4 [2,12].

## ANALYSIS

The analysis was performed with a finite element special purpose software, (FRANC2D) in 2D plain strain Condition [10,13]. The model was analyzed by dynamic relaxation solver, because of using contact element

[1] - United states Bureau of Reclamation
[2] - Spanish Instruction for Large Dams

in Rock/concrete interface, which was the only source of nonlinearity [13]. So, a linear elastic material model coupled to a linear elastic discrete fracture model was used in this analysis.
Although for crack propagation in quasi-brittle material nonlinear fracture Mechanics, (NLFM) should be used in large structures, such as dams, because of the smaller size of fracture process zone at the crack tip with respect to the structure size, limited errors should occur under the assumption of linear elastic fracture Mechanics [1,2,3,7,8].

**Tab. 4-** Load Cases

| Load Combinations | Load cases |
|---|---|
| Usual Load Combination | Self-weight + Hydrostatic Pressure at normal water level + Uplift Pressure (USBR Method)+ Internal Crack Pressure |
| Unusual Load Combination | Self-weight + Hydrostatic Pressure at maximum water level + Uplift Pressure (USBR Method)+ Internal Crack Pressure |
| Extraordinary Maximum Water Level | Self Weight + Hydrostatic Pressure at 5% more than crest level+ Uplift Pressure (USBR Method)+ Internal Crack Pressure |
| Earthquake Load Combination (Full reservoir) | Self-weight + Hydrostatic Pressure at normal water level + Uplift Pressure (USBR Method) + Inertial Forces induced by Earthquake (Pseudo-static Method) + Hydrodynamic Pressure |
| Earthquake Load Combination (empty reservoir) | Self-weight + Inertial Forces induced by Earthquake (Pseudo-static Method) |

In order to study the dam stability and simulate crack growth, initial non-cohesive cracks were placed at the region where, the critical stresses would occur (Figure 6).
Having specified the location of cracks, FRANC2D was able to automatically propagate the cracks and remesh around the cracks after each step [13]. The code computes stress Intensity factor (SIF) as crack propagation criteria on the base of LEFM and the kinking angle of the crack based on the maximum circumferential stress criteria. Prior to performing the analysis, it was necessary to specify the magnitude of crack increment and also the number of steps over which the crack would propagate. Finally, the SIF history versus crack length was plotted and crack stability was known comparing SIF with fracture toughness.

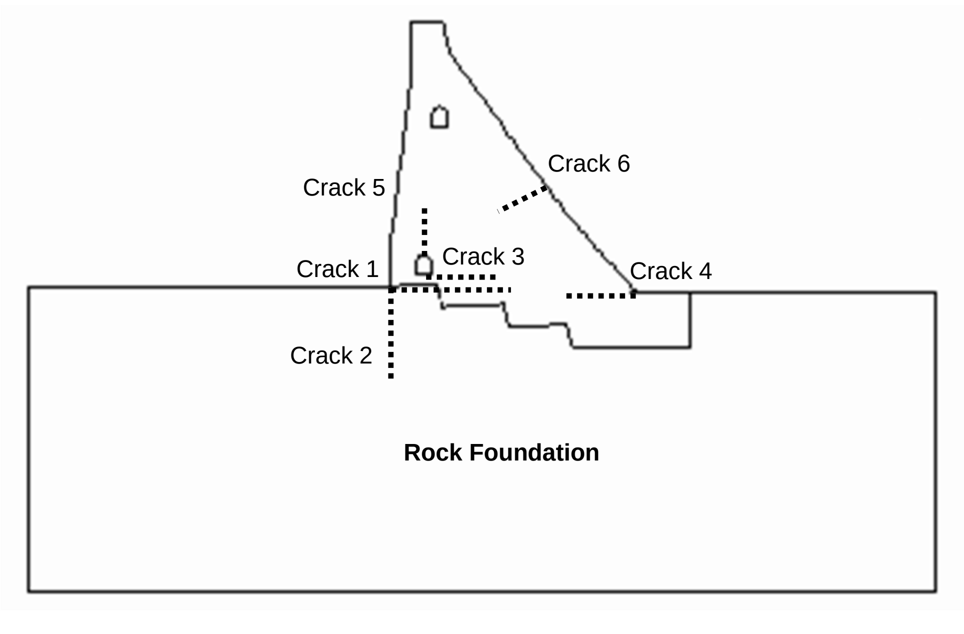



**Fig. 6-** Initial cracks in the dam body and foundation

## PRIMARY RESULTS

### First Load Combination (usual load)

Crack No.1 was placed at the upstream boundary of the dam/foundation interface. The crack growth was simulated over 10 steps of increment where each increment was 2 meters as suggested in Ref. [13]. Figure 7 shows the propagated crack No.1 and variation of KI with crack length, respectively.
The initial crack will propagate according max circumferential stress criteria by FRANC2D after F.E. analysis and finding the stress field around crack tip, considering crack increment that was defined by user. Then remeshing technique will be used and SIF's will determine with new crack profile and new F.E. analysis. This process will be continued until crack propagation be completed. We should note that the number of crack propagation steps will be defined by user and FRANC2D propagate the initial crack exactly according it without checking the LEFM criteria at each step. So we cannot determine the actual final crack length only considering the final crack shape. For this purpose we should use SIF variation which was plotted vs. crack length. According LEFM assumptions the initial crack will propagate when SIF exceeds the fracture toughness (KIc).

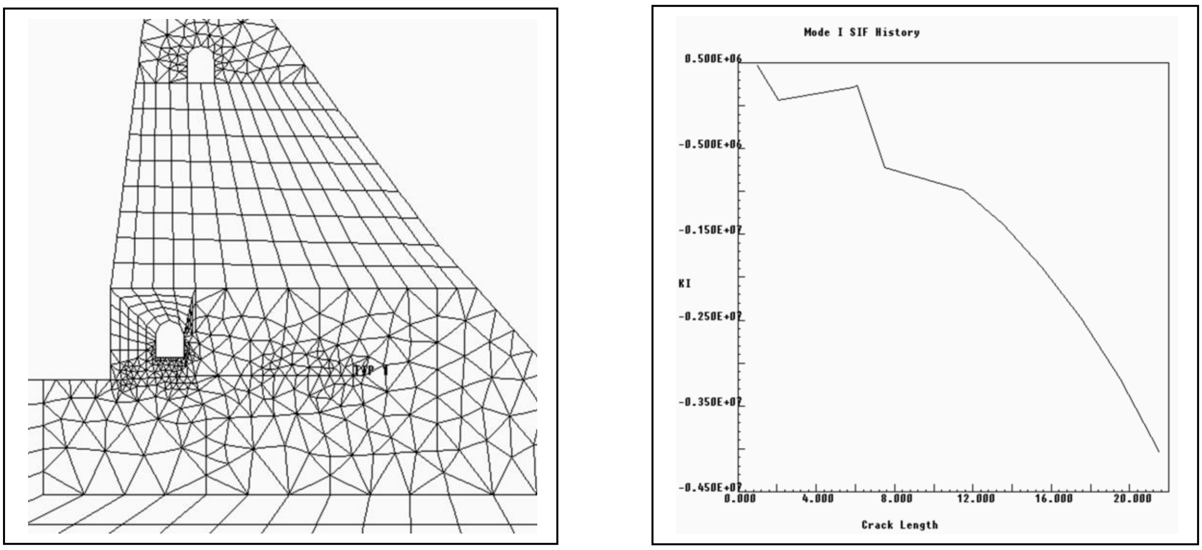



**Fig. 7-** Propagation of crack No.1 and its SIF history (usual load)

Table 4 shows the predicted final length of crack No.1 using the traditional criterion that crack propagation is in unstable manner when the KI reaches its critical value (KIc). So crack No.1 will propagate just 15% of the dam width.

**Tab. 5-** Final length of crack No.1 (usual load)

| Fracture Toughness of Interface | Final length of Crack No.1 |
|---|---|
| KIc=0.3E6 $Pa\sqrt{m}$ | 1.8m |
| KIc=0 | 6.2m |
| KIc=2.3E6 $Pa\sqrt{m}$ | Not Propagated |

Crack No.2 which was placed at the upstream heel, propagated over 10 steps by 2 meters increment toward the rock foundation. Figure 8 shows the propagated crack No.2 and SIF history, respectively. This crack will propagate 14.8m toward the rock foundation.

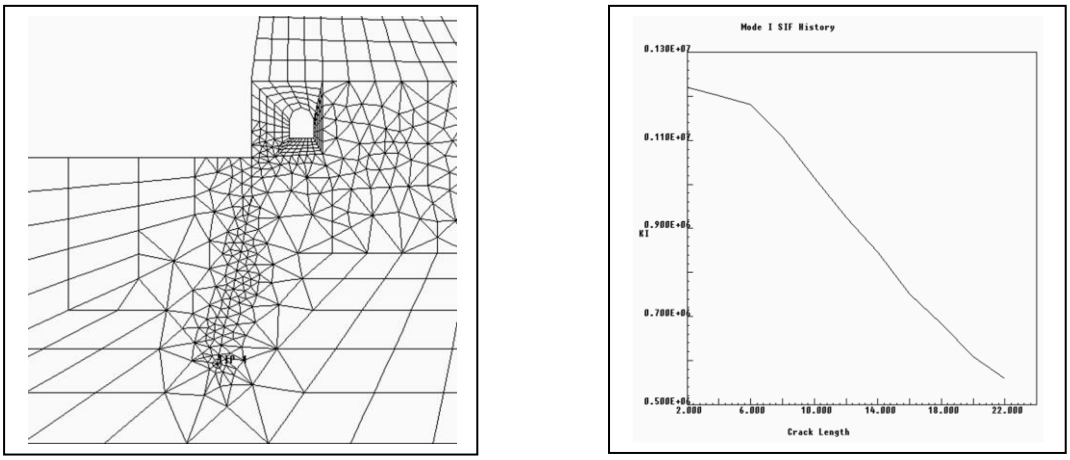



**Fig. 8-** Propagation of crack No.2 and its SIF history (usual load)

### Second Load Combination (unusual load)

In this situation crack No.1 predicted to propagate 20% of the dam width if the lowest fracture toughness is used as shown in Table 6.

**Tab. 6-** Final length of crack No.1 (unusual load)

| Fracture Toughness of Interface | Final length of Crack No.1 |
|---|---|
| KIc=0.3E6 $Pa\sqrt{m}$ | 6.8m |
| KIc=0 | 8m |
| KIc=2.3E6 $Pa\sqrt{m}$ | Not Propagated |

Crack No.2 propagated 16.5 meters toward the foundation. It was seen that the higher water level in reservoir causes more kinking of this crack.

### Third Load Combination (Extraordinary load)

This load case is similar to the two latest load cases. Cracks No.1 propagated as mentioned in Table 7 and crack No.2 propagated 19.6 meters toward the rock foundation.

**Tab. 7-** Final length of crack No.1 (Extraordinary load)

| Fracture Toughness of Interface | Final length of Crack No.1 |
|---|---|
| KIc=0.3E6 $Pa\sqrt{m}$ | 11.6m |
| KIc=0 | 14.5m |
| KIc=2.3E6 $Pa\sqrt{m}$ | Not Propagated |

### Forth Load Combination (Earthquake[1] in full reservoir)

In this load case, three expected initial cracks were studied. Crack No.1 at the upstream heel propagated as shown in Table 8. Table 8 indicated that crack No.1 was extended more than 2/3 of the base width. So, ZAVIN Dam will be unstable in this load case.

[1] - Magnitue of earth acceleration in the region of dam constructed was assumed according to the Iranian Earthquake Code (2800, Rev.2) [14]

**Tab. 8-** Final length of crack No.1 (Earthquake load in full reservoir)

| Fracture Toughness of Interface | Final length of Crack No.1 | |
|---|---|---|
| | Acceleration 0.2g | Acceleration 0.3g |
| KIc=0.3E6 $Pa\sqrt{m}$ | More than 25 meters | More than 25 meters |
| KIc=2.3E6 $Pa\sqrt{m}$ | More than 25 meters | More than 25 meters |
| KIc=0 $Pa\sqrt{m}$ | More than 25 meters | More than 25 meters |

Crack No.2 at the upstream was propagated toward the foundation and its final length would be more than 22 meters. Finally, crack No.3, which was placed at the bottom of the lower inspection gallery, propagated over 4 steps of 2 meters increment. Figure 9 illustrated the probable final profile of the crack and Table 9 shows the final length of this crack according SIF variation with crack length.

## Fifth Load Combination (Earthquake in empty reservoir)

In this case, regarding the critical direction of earthquake, three expected initial cracks were propagated.
Crack No.4 which was placed at the downstream toe of dam, propagated over 4 steps. Figure 10 illustrated the final profile of the crack and Table 10 shows its final length.

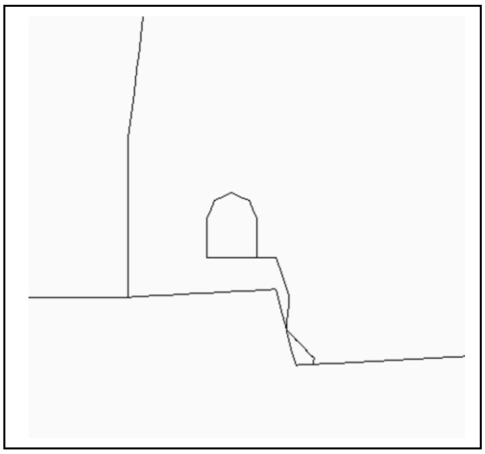
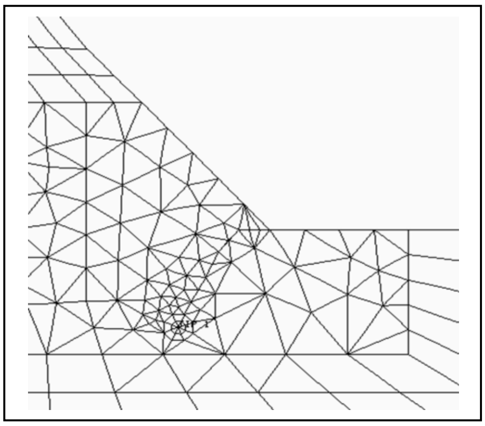


**Fig. 9-** Propagation of crack No.3 (Earthquake load in full reservoir)

**Tab. 9-** Final length of crack No.3 (Earthquake load in full reservoir)

| Fracture Toughness of Concrete | Final length of Crack No.3 | |
|---|---|---|
| | Acceleration 0.2g | Acceleration 0.3g |
| KIc=1.05E6 $Pa\sqrt{m}$ | Not propagated | Not propagated |
| KIc=2.3E6 $Pa\sqrt{m}$ | Not propagated | Not propagated |

Crack No.5 was placed at the downstream face of the dam, where the body slops changed. This crack propagated over 9 steps as show in Figure 11 and the final length of this crack is shown inTable11, according SIF variation with crack length.
Crack No.6 was placed at the roof of the upper inspection gallery and propagated over 5 steps as shown in Figure 12. Results indicated that SIF for this crack is lower than the concrete fracture toughness, so this crack is stable and would not propagate.



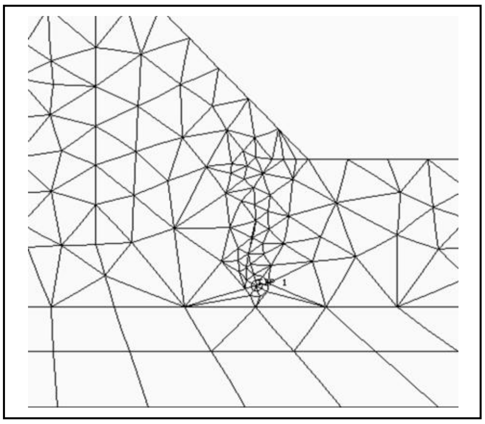
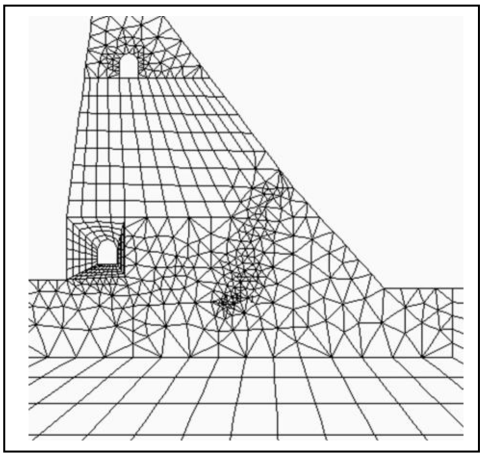


**Fig. 10-** Propagation of crack No.4 by earth acceleration 0.2g (left one) and acceleration 0.3g (right one), for earthquake load in empty reservoir

**Tab. 10-** Final length of crack No.4 (Earthquake load in empty reservoir)

| Fracture Toughness of Concrete | Final length of Crack No.4 | |
|---|---|---|
| | Acceleration 0.2g | Acceleration 0.3g |
| KIc=1.05E6 $Pa\sqrt{m}$ | Not propagated | Not propagated |
| KIc=2.3E6 $Pa\sqrt{m}$ | Not propagated | Not propagated |



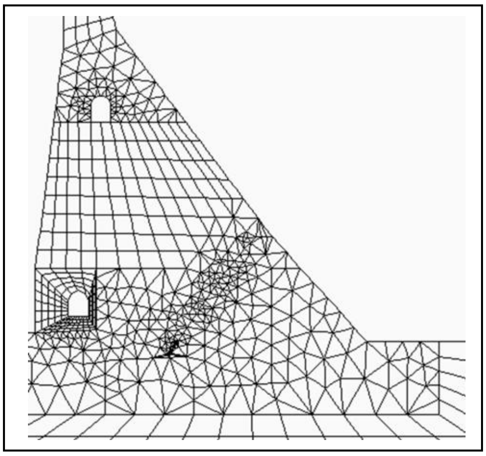


**Fig. 11-** Propagation of crack No.5 by earth acceleration 0.2g (left one) and acceleration 0.3g (right one), for earthquake load in empty reservoir

**Tab. 11-** Final length of crack No.5 (Earthquake load in empty reservoir)

| Fracture Toughness of Concrete | Final length of Crack No.5 | |
|---|---|---|
| | Acceleration 0.2g | Acceleration 0.3g |
| KIc=1.05E6 $Pa\sqrt{m}$ | Not propagated | 6m Crack Will Propagate to 14m |
| KIc=2.3E6 $Pa\sqrt{m}$ | Not propagated | Not propagated |

## CONCLUSION

An attempt was made to study the behavior of six predicted initial cracks lying at several places in the dam body and foundation of an existing concrete gravity dam "ZAVIN", using the concept of Linear Elastic Fracture Mechanics. The dam was analyzed by FRANC2D software under five load conditions and the following observations could be drawn:

1. In usual, unusual and extraordinary load combinations, the horizontal crack at the heel of dam will propagate 15%, 20% and 33% of the dam width, respectively.

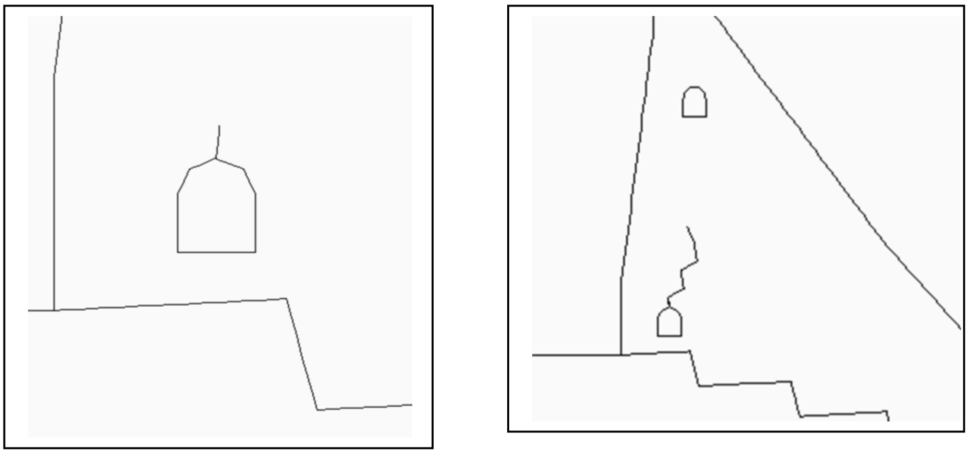


**Fig. 12-** Propagation of crack No.6 (earthquake load in empty reservoir)

2. In extreme load combination (empty reservoir), the initial cracks will not propagate. Therefore, *ZAVIN* dam is stable at this condition.
3. In extreme load combination (full reservoir), the initial heel crack will propagate more than 2/3 of the dam width, leading to complete separation of the dam base.
4. In extraordinary and extreme load combination (full reservoir), the initial vertical crack will propagate more than 25 meters toward the foundation. This major crack increases seepage from the reservoir and causes sliding instability.

## ACKNOWLEDGMENTS

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