Finite volume modelings of flow in T-junction open channels

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Abstract

In this paper, we consider numerical modeling of flow in 90° intersections of open channels. The modeling is based on finite volume solution of the shallow water equations (St. Vnant equations). The method used is upwind based with first order accuracy in space and time. For networking of solution space the unstructured triangular cells have been used. In numerical tests, flow parameters such as discharge distribution in channels, length of the flow separation zone, and possible jump locations have been obtained. The comparison of the results with experimental data shows the proper prediction ability of the numerical method used. Using the numerical effort applied, we investigated the flow distribution in channels based on inflow Froude number and the roughness effect on vortex length in lateral branch.

Key words: Channel intersections, Finite volume method, Flow division, Shallow water equations.

1. Introduction

Severe storms pass through urban areas could lead to flooding in the streets. Proper prediction of the flow conditions will prevent material losses and reduce the risk of construction of structures associated with fluid flow. Hence, study of the flow characteristics at channels intersection and discharge distribution is equally important for the design of an open channel network and flood protection. Experimental studies of flow field due to equipment limitation, being costly and time consuming, and also because of the problems of lack of similarity with real flow field, less used. In numerical modeling, flow conditions and dimensions can be changed easily and with low cost, so we can obtain different design results.

Taylor (1944) first addressed the topic of open-channel junction flow by focusing on the depth ratio between the upstream branches and the downstream channel. The results are a momentum analysis that yields a predictive equation for the depth ratio. Taylor's paper is important for its identification of the need of theoretical description of the open-channel junction and the groundwork it formed for future investigations. One of the first studies of confluent supercritical flow in a three-channel junction was reported by Bowers (1950) who showed that the formation of hydraulic jumps in the inlet channels of the junction depends on the junction geometry and the upstream flow rates. He found that the position of the jump is dependent upon the flow rates in the incoming channels and when a jump does not form, waves are created at the junction. He also observed that decreasing the discharge in the main inlet channel causes the jump in that channel to move upstream and causes the jump in the side channel to move downstream. Law and Reynolds (1966) concluded that the momentum equation, with a proper assumption of hydraulic pressure force on the depth-averaged stagnation dividing streamline surface, and the energy consideration both describe the one-dimensional flow characteristics. Ramamurthy and Satish (1988) found that, for a short branch channel with the downstream Froude number exceeding a threshold value, the branch flow exhibits an unsubmerged recirculation region. They assumed critical flow at the maximum width-contracted section of the recirculation region to derive a relationship for downstream-to-upstream depth ratio, downstream-to-upstream discharge ratio, and upstream Froude number. Ramamurthy et al. (1990), skipping the assumption of critical condition, assumed no energy loss along the depth-averaged stagnation dividing streamline surface to obtain an expression for the momentum transfer rate from the main to the branch channel.

Neary and Odgaard (1993) examined the effects of bed roughness on the three-dimensional structure of a dividing flow and considered the flow being similar to a bend flow. Hsu et al. (2002) established a depth-discharge relationship and an energy-loss coefficient derived from the energy equation. Ramamurthy et al. (2007) developed a 3D k-ε model with free surface tracking capability to study dividing open-channel flows. El Kadi Abderrezzak et al. (2011) study the characteristics of dividing critical flows in a 90°open-channel junction. They identified four main flow patterns considering the location and length of the hydraulic jumps that develop across the main and lateral channels.

In the present study, the Roe finite volume scheme is used for modeling flow at a T-junction. To ensure the performance of the numerical model, the results are compared with experimental results of other researchers. Flow details including the recirculation zones and hydraulic jumpings will also be studied.

2. Mathematical Model

The 2D equations of depth-averaged shallow water with source term can be expressed in the following vectorial conservative form:

$$\frac{\partial W}{\partial t}+\frac{\partial F_x}{\partial x}+\frac{\partial F_y}{\partial y}=\begin{pmatrix} 0 \\ -\dfrac{\tau_{b,x}}{\rho} \\ -\dfrac{\tau_{b,y}}{\rho} \end{pmatrix} \tag{1}$$

In which

$$w=\begin{pmatrix} h \\ q_x \\ q_y \end{pmatrix} \quad ; \quad F_x=\begin{pmatrix} q_x \\ \dfrac{q_x^2}{h}+\dfrac{gh^2}{2} \\ \dfrac{q_x q_y}{h} \end{pmatrix} \quad ; \quad F_y=\begin{pmatrix} q_y \\ \dfrac{q_x q_y}{h} \\ \dfrac{q_y^2}{h}+\dfrac{gh^2}{2} \end{pmatrix} \tag{2}$$

where q_x and q_y are unit discharges in the *x* and *y* directions, respectively; *h* is the water depth; g is the gravity acceleration; $\tau_{b,x}$ and $\tau_{b,y}$ are the bed shear stresses in the *x* and *y* directions, and ρ is the fluid density. In this study, the bed slope source terms are estimated by using the manning's formula:

$$G = \begin{pmatrix} 0 \\ -c_f U_x |U_x| \\ -c_f U_y |U_y| \end{pmatrix} \qquad ; \qquad c_f = \frac{gn^2}{h^{1/3}} \tag{3}$$

where U_x and U_y are the velocity of flow in the *x* and *y* directions, respectively, and *n* is the Manning's roughness coefficient.

3. Numerical Scheme

Using time discretization and simplification of the Eq. 1, we could obtain the following equation:

$$W^{n+1} = W^n - \Delta t \left(\frac{\partial F_x}{\partial x}(W^n) + \frac{\partial F_y}{\partial y}(W^n) \right) + \Delta t G^n \tag{4}$$

In above equations W^n is the vector of conserved variables at time t^n with the time step of Δt. An upwind model can be performed for spatial discrerization. By integrating the Eq. 1 over a cell *i* with area of A_i and application of Gauss divergence theorem, the following equation will be achieved:

$$A_i \frac{W_i^{n+1} - W_i^n}{\Delta t} + \int_{L_i} \left(F_x \tilde{n}_x + F_y \tilde{n}_y \right) dL = \int_{c_i} G_k dA \tag{5}$$

Where $\tilde{n} = (\tilde{n}_x, \tilde{n}_y)$ is a unit vector normal the cell face, and L_i is the cell boundary. On the left hand side of the equation (5), the second term is as follow:

$$\int_{L_i} \left(F_x \tilde{n}_x + F_y \tilde{n}_y \right) dL = \sum_{j \varepsilon k_i} \int_{L_{ij}} \left(F_x \tilde{n}_x + F_y \tilde{n}_y \right) dL = \sum_{j \varepsilon k_i} \phi_{ij} \left(W_i, W_j, n_{ij} \right) \tag{6}$$

Where L_{ij} is the length of cell face *ij* (face between cells *i* and *j*), K_i is the number of faces at cell *i* (for a triangular cell K_i would be 3), ϕ_{ij} is the numerical flux at the cell face *ij*, and $n_{ij} = (n_x, n_y)_{ij}$ is the normal vector at the cell face *ij*. The numerical flux in upwind method would be calculated as follow:

$$\phi_{ij} = \frac{Z(W_i, n_{ij}) + Z(W_j, n_{ij})}{2} - \frac{1}{2}\left|\phi(W_i, W_j, n_{ij})\right|(W_j - W_i)$$
$$|\phi| = X|D|X^{-1} \quad ; \quad Z = F_x n_x + F_y n_y \tag{7}$$
$$|D| = \begin{bmatrix} |\bar{\lambda}_1| & 0 & 0 \\ 0 & |\bar{\lambda}_2| & 0 \\ 0 & 0 & |\bar{\lambda}_3| \end{bmatrix} \quad ; \quad X = \begin{bmatrix} 0 & 1 & 1 \\ -\tilde{n}_y & \bar{U}_x + \bar{C}\tilde{n}_x & \bar{U}_x - \bar{C}\tilde{n}_x \\ \tilde{n}_x & \bar{U}_y + \bar{C}\tilde{n}_y & \bar{U}_y - \bar{C}\tilde{n}_y \end{bmatrix}$$
$$\bar{\lambda}_1 = n_x \bar{U}_x + n_y \bar{U}_y \quad ; \quad \bar{\lambda}_2 = \bar{\lambda}_1 + \bar{C}L_{ij} \quad ; \quad \bar{\lambda}_3 = \bar{\lambda}_1 - \bar{C}L_{ij}$$

For defining the average values in the first order scheme of Roe at each cell, the following equations may be used:

$$\bar{U}_x = \frac{\sqrt{h_i}U_{x,i} + \sqrt{h_j}U_{x,j}}{\sqrt{h_i} + \sqrt{h_j}} \quad , \quad \bar{U}_x = \frac{\sqrt{h_i}U_{y,i} + \sqrt{h_j}U_{y,j}}{\sqrt{h_i} + \sqrt{h_j}} \quad , \quad \bar{C} = \sqrt{g\frac{h_i + h_j}{2}} \tag{8}$$

Where h_i is the depth of water in cell *i*, and $U_{x,i}$ and $U_{y,i}$ are flow velocities, respectively in *x* and *y* directions in cell *i*.

4. Numerical method application

In order to evaluation of the numerical method, flow in T-junction in channels has been simulated as shown in Fig. 1. The channels have equal width of o.3m, and the length of the main channel and the side branch respectively considered 4.5m and 1.2 m and the solution domain was divided into 5388 unstructured triangular cells. All the channels are horizontal and made of glass. The Manning roughness coefficient was n=0.0083.

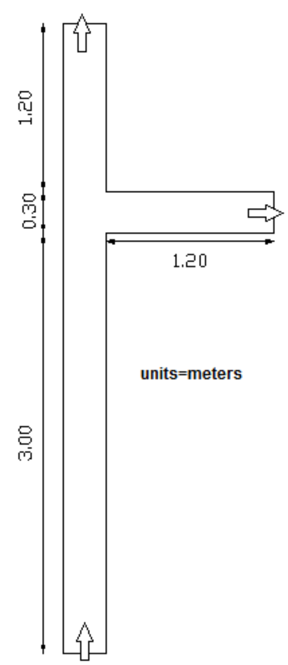

Fig 1: Schematic layout of the solution space.

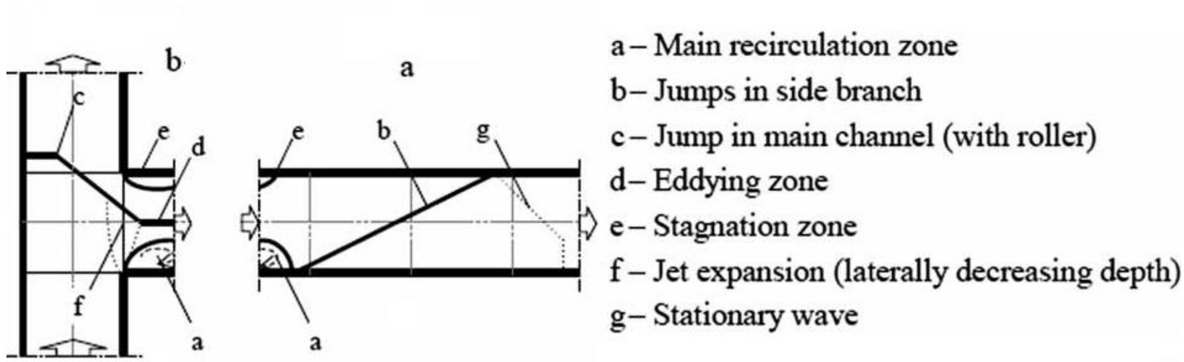

Fig 2: Flow patterns at junction vicinity a) lateral branch b) main channel.

El Kadi Abderrezzak et al. presented a flow patterns at junction vicinity(Fig. 2). Fig. 2(a) shows an oblique jump develops downstream of the recirculation zone reflecting steady waves on the walls. Also, Fig. 2(b) shows a hydraulic jump occupies the entire width.

In numerical analysis the constant flow discharge was 12lit/s. In the case of a subcritical flow in upstream, two boundary conditions are needed, so that *u* and *v* are directly given and h is obtained. At the downstream end, all physical variables, *u*, *v*, and *h* at the boundary face are the same as the internal variables. A free slip condition is applied at the solid boundary, i.e., the normal velocity component at the face is set to zero. In Fig. 3, flow depths are plotted. Comparing the figure with flow pattern provided by El Kadi Abderrezzak et al. (Fig. 2), it is clear that the numerical solution performed well showing the vortex and jump in lateral branch and also jump in main channel and matched the presented pattern. The contours of streamlines are plotted in Fig. 4. It can be seen that the vortex formed at the beginning of the lateral branch is consistent with the flow patterns presented in Fig. 2. The vortex length for this flow was measured 0.3 m. Fig. 5 shows contour plots for the velocity component *u*. We could see that velocity at the beginning of the lateral branch is negative which indicates the vortex zone.

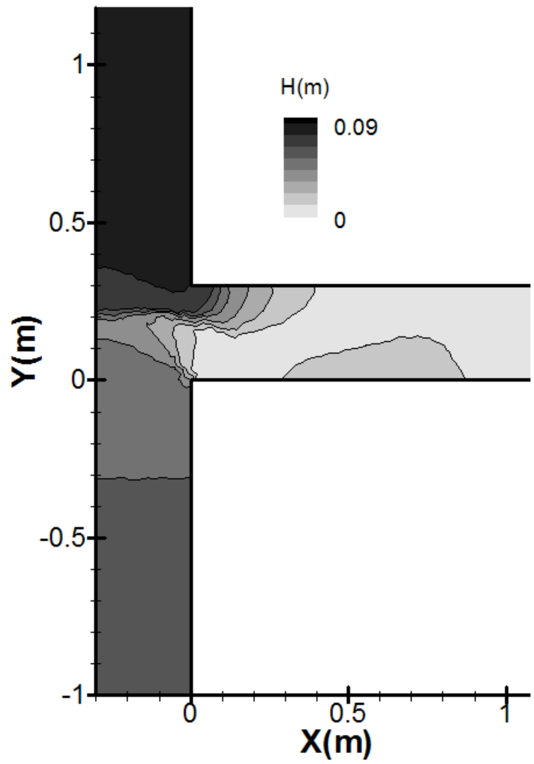

Fig 3: Flow depth Contours.

The comparison of numerical model results with experimental results presented by El Kadi Abderrezzak et al. for the input discharge of 12 l/s, is given in Fig. 6. As seen in this Figure, the numerical modeling simulated well the jump at the lateral branch and the junction.

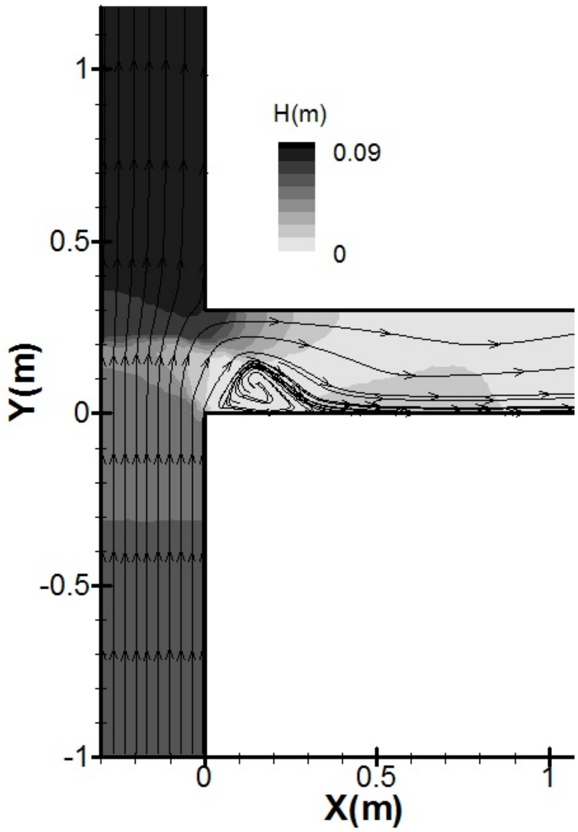

Fig 4: Streamlines and flow depth Contours.

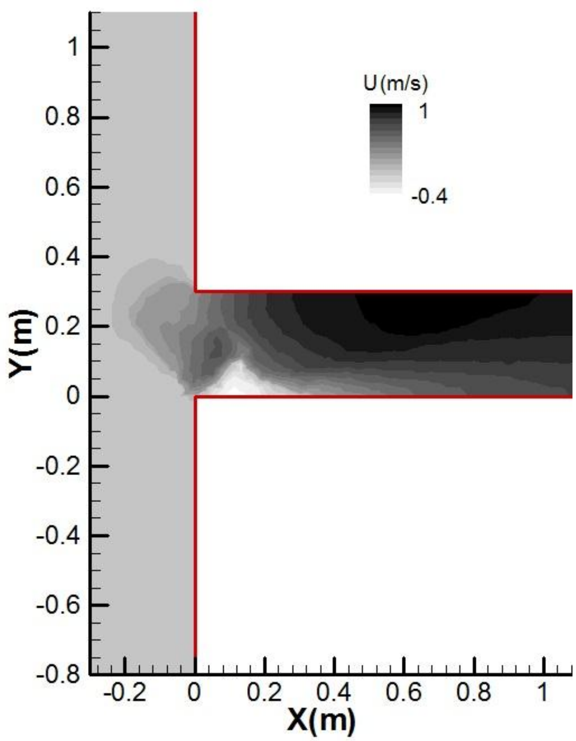

Fig 5: Contours for velocity *u*.

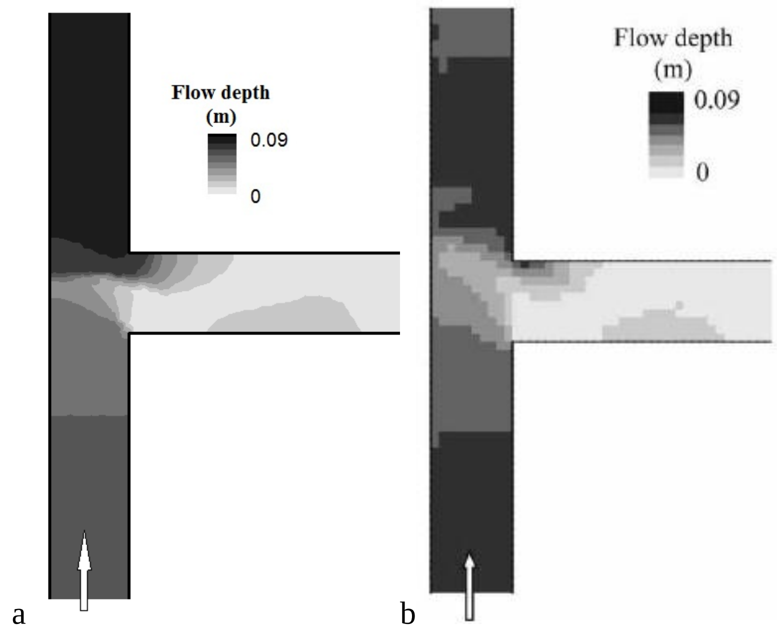

Fig 6: Contours of flow depth a) Numerical results b) experimental results (El Kadi Abderrezzak et al.).

For evaluation of the effect of the bed friction source term, we simulated the channel in accordance with mentioned characteristic (Fig. 1), by application of 5 different roughness coefficients. Variation of the vortex length at the lateral channel was plotted against the roughness coefficient in Fig. 7. It is obvious that increasing in roughness coefficient increases the vortex length. It should be noted that changing the roughness coefficient has no serious effect on output discharge rate of the branch.

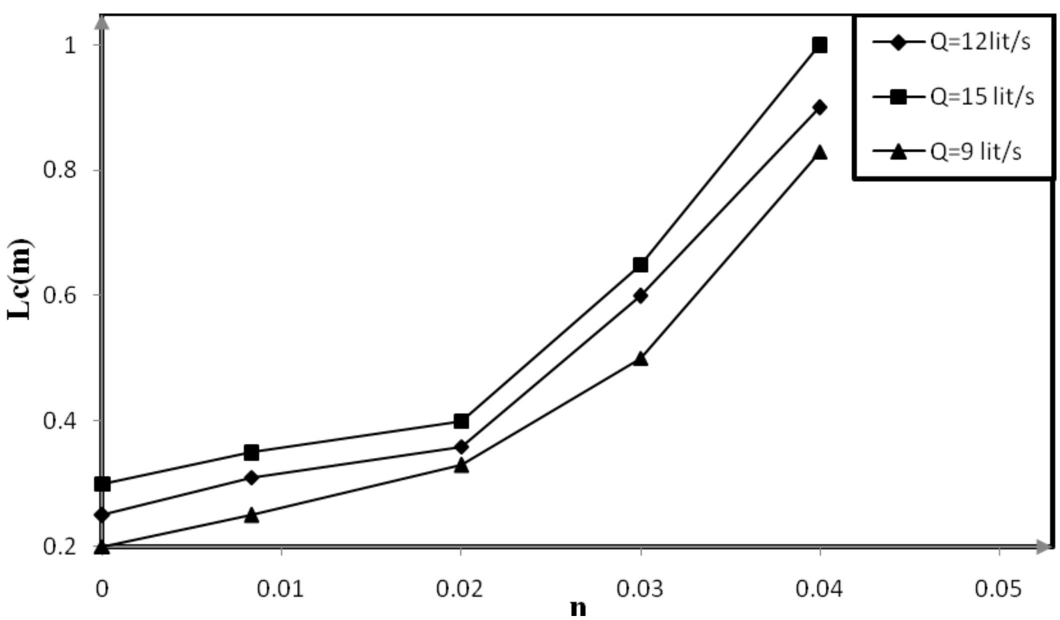

Fig 6: The effect of roughness coefficients.

5. Conclusions

In this paper, the shallow water flow at the 90 ° intersection in open channels was modeled. The Roe finite volume method at unstructured triangular cells has been used for this purpose. Bed friction was simulated using Manning equation. Flow characteristics at channels intersection such as flow separation area and location of the hydraulic jump, were obtained and compared with available experimental results. This comparison showed the proper performance of the numerical model. The examination of roughness effect on flow in channels intersection indicated that increasing the roughness coefficient of the channels will increase vortex length at the lateral branch.

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