Estimation of Creep Deformations in Circular Tunnels Considering Elasto-Plastic Behavior

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Abstract

By excavating an underground space, the state of stress and displacement in the surrounding medium are changed in comparison to the initial state. As time passes, the variation of the displacement mainly depends on the creep behavior of the hosting rock mass. In this paper, an elasto-viscoplastic creep model is proposed. In the proposed model, the main purpose is to consider plastic deformations increasing with time. The viscoplastic behavior of rocks plays a key role in the tunneling works, especially for deep tunnels subjected to large in situ stresses. Using non-linear Hoek-Brown yield criterion in a creep model is the other important aim of this paper, which eliminates estimating specific equivalent Mohr-Coulomb strength parameters from the Hoek-Brown parameters. The equations related to the proposed model are derived and then, to reach a numerical solution of the equations, the finite difference software (FLAC^{2D}-FISH Editor) is used. The application of the proposed model is illustrated through an example analyzed numerically using the finite difference software FLAC^{2D}.

Key words: Creep, Rock Mass, Tunnel, Elasto-Viscoplastic model, Hoek-Brown Criterion

1-Introduction

Time-dependent responses of rock materials should be taken into account in many engineering problems, such as long-term stability analysis of underground constructions (Zhou et al., 2008). The tunnels excavated in rocks would considerably experience deformations increasing with time, that might lead to a delayed failure of structures. In the distant past, for the analysis of time-dependent behavior of tunnels, especially creep, linear creep models were used (Jaeger and Cook, 1969). However, with such models the experimental results do not agree with the theoretically obtained curves (Valsangkar and Gokhale, 1971). Nowadays, the classic viscoelastic Burger model and the classic Burger-MC model are widely used by engineers to predict time-dependent behavior of rock especially in tunneling works. Besides, these two models are practical models which exist in the finite difference software FLAC^{2D}. In this paper, with respect to powers and weaknesses of previous creep models and based on the classic Burger-MC model, an elasto-viscoplastic creep model is proposed. In the proposed model both viscoelastic and viscoplastic deformations are considered and non-linear Hoek-Brown yield criterion is used. Although the proposed model is not included directly in the FLAC^{2D}'s constitutive laws library, alternatively, the programmable language FISH (FLACish) helped to implement it indirectly in the medium of the software. For verification of the proposed model, the model is implemented in a circular tunnel (engineering instance) and then, creep diagram of the model and results of numerical analysis are compared with some practical models which exist in FLAC^{2D}.

1. The elasto-viscoplastic model

2.1 The classic Burger-MC model

The classic Burger-MC model consists of a Burger body, which is able to present the primary and secondary creep regions of rock masses (Fahimifar et al., 2010), is connected in series with a plastic Mohr-Coulomb unit. The constitutive laws of the classic Burger-MC model are characterized by an elastoplastic volumetric behavior and a viscoelasto plastic deviatoric behavior. The deviatoric behavior is schematically illustrated in Fig. 1.

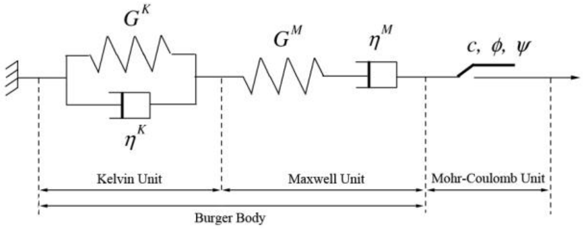

Fig. 1. Schematical representation of Burger-MC creep model

In the classic Burger-MC model a Kelvin unit is characterized by its shear modulus G^K and viscosity η^K , a Maxwell unit is characterized by its shear modulus G^M and viscosity η^M and a plastic Mohr-Coulomb unit, is characterized by its cohesion c, friction angle ϕ and dilation angle ψ .

2.2 The elasto-viscoplastic model

In order to model creep behavior of rocks, many researchers have been used the slider element in parallel with the viscous element to predict viscoplastic deformations of rocks

(Tomanovic, 2006; Sterpi and Gioda, 2007; Malan, 1999). Inasmuch as, creep deformation of rocks is considered to be both reversible and irreversible, in this paper a new elasto-viscoplastic creep model is proposed which considers immediate deformations, reversible and irreversible deformations increasing with time. In the proposed model the Burger body (a Kelvin and a Maxwell unit in series) is connected in series with a viscoplastic unit. The viscoplastic unit consists of a dashpot element which is connected in parallel with a plastic Hoek-Brown slider element. The constitutive laws of the proposed model are characterized by an elastoplastic volumetric behavior and an elasto-viscoplastic deviatoric behavior. The deviatoric behavior of the proposed model is schematically illustrated in Fig. 2.

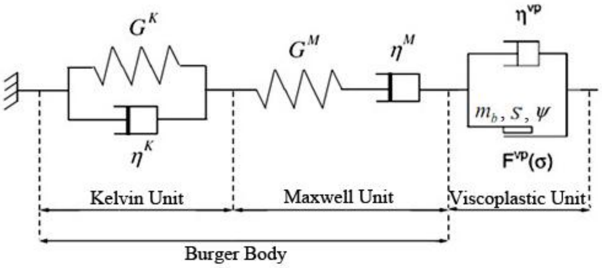

Fig. 2. Schematical representation of the proposed model

In the proposed model a Kelvin unit is characterized by its shear modulus G^K and viscosity η^K , a Maxwell unit is characterized by its shear modulus G^M and viscosity η^M and a viscoplastic unit consists of a dashpot which is characterized by its viscosity η^{vp} and a plastic Hoek-Brown slider which is characterized by its strength parameters m_b , S and the dilation angle ψ . The constitutive equations of the proposed model can be worked out as follows:

Total strain rate:

$$\dot{\varepsilon}^t_{ij} = \dot{\varepsilon}^M_{ij} + \dot{\varepsilon}^K_{ij} + \dot{\varepsilon}^{vp}_{ij} \tag{1}$$

Introducing the total strain rate $\dot{\varepsilon}^t_{ij}$, Maxwell strain rate $\dot{\varepsilon}^M_{ij}$, Kelvin strain rate $\dot{\varepsilon}^K_{ij}$ and the viscoplastic strain rate $\dot{\varepsilon}^{vp}_{ij}$. The Maxwell and Kelvin units do not carry volumetric stress and they only participate in carrying deviatoric stress. The amount of the viscoplastic strain rate is described by the flow rule governed by the plastic slider (Sterpi and Gioda, 2007):

$$\dot{\varepsilon}^{vp}_{ij} = \dot{\lambda}\frac{\partial Q}{\partial \sigma_{ij}} \tag{2}$$

The deviatoric behavior of the proposed model can be formulated as:

Total strain rate:

$$\dot{e}^t_{ij} = \dot{e}^M_{ij} + \dot{e}^K_{ij} + \dot{e}^{vp}_{ij} \tag{3}$$

Maxwell unit:

$$\dot{e}^M_{ij} = \frac{\dot{S}_{ij}}{2G^M} + \frac{S_{ij}}{2\eta^M} \tag{4}$$

Kelvin unit:

$$S_{ij} = 2\eta^K \dot{e}^K_{ij} + 2G^K e^K_{ij} \tag{5}$$

Dashpot:

$$\dot{e}_{ij}^{vp} = \frac{S_{ij}^{vp}}{2\eta^{vp}} \qquad (6)$$

Plastic Hoek-Brown slider:

$$\dot{e}_{ij}^{p} = \lambda\frac{\partial Q}{\partial\sigma_{ij}} - \frac{\dot{e}_{vol}^{p}}{3}\delta_{ij} \qquad (7)$$

$$\dot{e}_{vol}^{p} = \dot{\lambda}[\frac{\partial Q}{\partial\sigma_{11}} + \frac{\partial Q}{\partial\sigma_{22}} + \frac{\partial Q}{\partial\sigma_{33}}]$$

And the volumetric behavior is given by:

$$\dot{\sigma}_0 = \mathrm{K}(\dot{e}_{vol} - \dot{e}_{vol}^{p}) \qquad (8)$$

In the above equations, the symbols s_{ij} and e_{ij} are the deviatoric stress and the deviatoric strain, respectively, K is the bulk modulus, Q is the plastic potential, λ is a multiplier and δ_{ij} is the kronoker parameter which is equal to 1 in a condition of $i=j$, otherwise it is equal to 0. The superscripts K and M refer to the Kelvin and Maxwell components of the corresponding variables, and the superposed dot denotes time derivative. $\dot{e}_{ij}^{p}$ and $\dot{e}_{vol}^{p}$ are the deviatoric and volumetric plastic strain rates respectively, and σ_{ij} is the total stress. For the plastic Hoek-Brown slider, the yield criterion f and the plastic potential Q with respect to non-associated flow rule are formulated as:

$$f = \sigma_1 - \sigma_3 + \sqrt{-m_b\sigma_{ci}\sigma_3 + S\sigma_{ci}^2} \qquad (9)$$

$$\mathrm{Q} = \sigma_1 - \sigma_3 N_{\psi} \qquad (10)$$

Where m_b and S are the strength parameters, σ_{ci} is the uniaxial compressive strength of the intact rock material, ψ is the material dilation, σ_1 and σ_3 are the minimum and maximum principal stresses (compression negative), $N_{\psi} = (1 + sin\psi)/(1 - sin\psi)$.

The proposed model which is illustrated in figure 2, is implemented in finite difference software FLAC^{2D} using the built-in FISH language for constitutive models. The programmable language FISH offers an environment to user to introduce a new function or constitutive model that could work in such a way that the built-in constitutive models could. The FISH function calculates the value of c and ϕ for each zone in every step. Thus, as σ_3 changes, the values of c and ϕ will also change. It is noted that the instantaneous values of c and ϕ calculated in this way closely match those calculated using Hoek's (Hoek, 1990) expressions based on normal and shear stress (Itasca FLAC^{2D}, 2004). Finally by application of the plastic yield condition (f = 0) , the parameter $\dot{\lambda}$ will be determined as:

$$\dot{\lambda} = \frac{\sigma_1(1 - sin\phi_i) - \sigma_3(1 - sin\phi_i) + 2C_i cos\phi_i}{2\eta^{vp}(1 + N_{\psi} + \frac{N_{\psi}}{3}sin\phi_i - \frac{sin\phi_i}{3})} \qquad (11)$$

Where C_i and ϕ_i are the instantaneous values of C and ϕ. Now that the $\dot{\lambda}$ is computed, the viscoplastic strain rate component can be computed by the flow rule described in Eq.7:

$$\dot{\varepsilon}_1^{vp} = \dot{\lambda} \qquad (12)$$

$$\dot{\varepsilon}_1^{vp} = 0 \qquad (13)$$

$$\dot{\varepsilon}_1^{vp} = -\dot{\lambda}N_{\psi} \qquad (14)$$

The governing equations of the constitutive model are implemented in numerical software FLAC^{2D} using the built-in language FISH function for the constitutive model. As long as the stress level is below the threshold described by the slider element, the proposed model behaves like the viscoelastic Burger model. For the higher stress level where the plastic limit condition is satisfied, the viscoplastic unit calculates the plastic strain which is added to the elastic Burger strain resulting total elasto-viscoplastic strain.

2. Results and Analysis

The potential applicability of the proposed model is tested through an example (Sterpi and Gioda, 2007) in the medium of the finite difference software FLAC^{2D}. The problem concerns the full face excavation, at a rate of 6m/day, of a circular tunnel of radius R =4.5 m, in a homogeneous, isotropic rock mass, subjected to an isotropic state of stress σ_0=10 MPa. This corresponds to a tunnel depth of approximately 400 m. Creep parameters of the hosting rock mass and the strength parameters of rock are presented in Table 1. and Table 2. respectively. The numerical analysis considers a creep time of 365 days.

Creep parameters	Value
K(MPa)	1470
G^M (MPa)	678
η^M (Mpa.day)	321555
G^K (MPa)	5848
η^K (Mpa.day)	49470
η^{vp}(Mpa.day)	1651

Table 1. Creep parameters of rock mass used in the present study

K: the Bulk modulus, G^M: the Maxwell shear modulus, η^M: the Maxwell viscosity , G^K: the Kelvin shear modulus, η^K: the Kelvin viscosity, η^{vp}: the viscoplastic viscosity.

Strength parameters	Value
GSI	50
m_i	12
σ_{cm} (MPa)	13
m_b	2.01
S	0.004
ψ	15.4

Table 2. Strength parameters of rock used in the present study

GSI: Geological Strength Index, m_i: the Hoek-Brown strength parameter of intact rock, σ_{cm}: the rock mass compressive strength, m_b: the Hoek-Brown strength parameter of rock mass, S: the Hoek-Brown strength parameter of rock mass, ψ: the dilation angle.

Fig. 3. shows the radial convergence of the tunnel face using the Burger, Burger-MC and the proposed model.

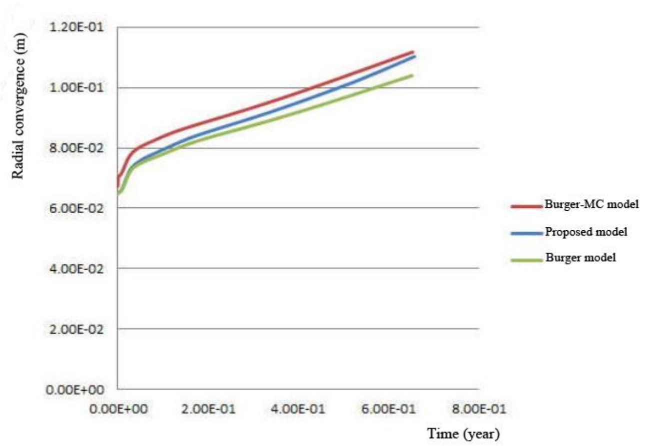

Fig. 3. Comparison between the Burger, Burger-MC and the proposed model for the radial convergence of the tunnel face

It can be seen in Fig. 3. that at early times the radial convergence of the tunnel face produced by the proposed model is near to the Burger's model and as time passes by increasing viscoplastic (time-dependent irreversible) deformations it reaches to the Burger-MC's model. Inasmuch as, elastic and plastic deformation of rocks are both immediate and time-dependent features, the results of creep behavior of the proposed model are in proper agreement to the time-dependent behavior of rock masses in a real medium.

3. Conclusions

In this study, based on the well-known Burger-MC model, a new elasto-viscoplastic creep model is proposed. The proposed model consists of a Burger body which is connected in series with a viscoplastic unit accounting for the irreversible creep deformation of rock. Besides, in the proposed model the slider element satisfies the non-linear Hoek-Brown yield criterion which is used most commonly to characterize failure of rock masses in tunneling projects. The equations of the proposed model are implemented in numerical software FLAC^{2D} using the built-in language FISH function for the constitutive model. Comparative analysis between the creep deformation of the proposed model and that of the Burger and Burger-MC models indicates that the proposed model can describe elastic and plastic creep deformation of rock as two stages: the primary elastic creep deformation and the secondary creep including both elastic and plastic with constant strain rate. It is also observed that the irreversible creep deformation is crucial in the prediction of time-dependent behavior of rock especially at high stress level where the rock gets its plastic phase.

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