Analytical Solution of Kinematic Response of Batter Pile under P and SV Waves Earthquake

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Abstract

This paper develops an analytical solution to investigate the kinematic response of a batter pile subjected to vertical and horizontal earthquake components (P and SV waves). The pile is assumed to have a circular cross section and perfectly attached to the soil. In addition, the soil layer is assumed to be homogenous and elastic. The governing differential equation of the soil-pile system is first derived and solved using appropriate boundary conditions. The kinematic response of the batter pile is obtained considering stress free condition at the pile head and the ground surface. The effects of contributing parameters on the pile response such as batter angle, pile geometry, and soil profile properties are considered and the results will be presented. It will be shown that the bending moment, axial and horizontal forces and displacement of batter pile head with the free head surface of soil will be induced in the pile due to the components earthquake excitation. In addition, the ratio of vertical and horizontal pile kinematic response with soil surface is changed with variation of batter angle.

Keywords: Batter pile, kinematic response, vertical and horizontal earthquake excitations, elasto-dynamic.

1. Introduction

Vertical piles are used in foundations to carry vertical and lateral loads. When the horizontal load per pile exceeds the value suitable for vertical piles, battered piles are used along with vertical piles to enhance the foundation lateral stiffness as well as capacity. Applications of batter piles include offshore structures, bridges, and towers. These types of structures are usually subjected to overturning moment due to wind, waves and earthquake excitation. The degree of batter is the angle between the pile axes with the vertical direction and can be as high as 30°. The response of piled foundations subjected to earthquake loading in the horizontal and vertical direction depends largely on piles, surrounding soil, and soil-pile interaction. It is well known that the seismic excitation transmitted to the base of a pile-supported structure is usually smaller than the free-field motion, because of the dynamic interaction between the foundation and the surrounding soil. This interaction develops even in the absence of a superstructure and is referred to as kinematic effect (Fiq. 1). The kinematic response for vertical piles has been well identified by many researchers, for example Kaynia and Kausel, 1982; Fan et al., 1991; Gazetas and Makris, 1992; Gazetas and Mylonakis, 1998. However, for pile response under vertical direction, there is a rare study reported in the literature. Mylonakis and Gazetas, 2000; studied the kinematic response of a single load free vertical pile subjected to the vertical earthquake component.

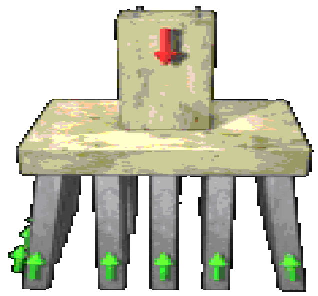


Fig 1. Application of batter pile for support of bridge piers

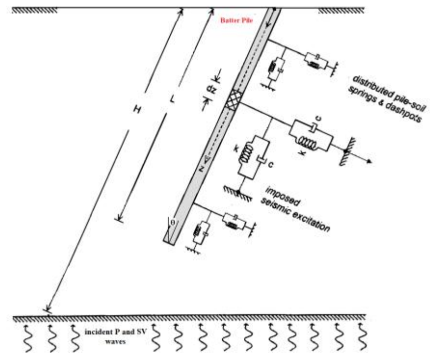


Fig 2. Batter pile under vertical and horizontal earthquake excitation

In addition, several studies on the seismic response of batter piles are investigated by (Guin, 1997). More recently, several researchers examined the seismic response of batter piles (Sadek and Shahrour, 2006; Poulos, 2006). Results from these studies indicate that little coupling exists between vertical and horizontal motions, and that uncoupled analyses are realistic for control motions.

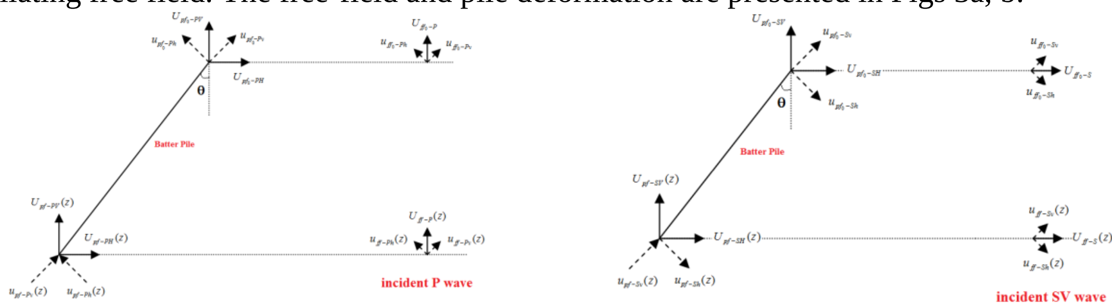
In this paper, the response of a battered pile with closed form solution to vertical and horizontal earthquake components (P and SV waves) is investigated. In other words, influence battered angle on the manner of pile under vertical and horizontal excitation will be studied.

2. PROBLEM DEFINITION

The problem treated in this study is shown in Fig. 2. A single battered pile embedded in a homogeneous soil layer resting on rigid bedrock, subjected to vertical and horizontal seismic excitation. The soil is assumed to be elastic with thickness H, Young's modulus E_s, mass density ρ_s, shear wave velocity V_s, compressional waves V_p, soil Poisson's ratio ν and linear hysteretic damping β. The pile is a solid cylinder of length L, diameter d, and Young's modulus E_P, pile cross sectional area A_P, second moment of area I_P, mass density ρ_P and the seismic loading under cyclic vibrational frequency, ω. The excitation consists of vertically propagating harmonic compressional and shear waves imposed at the base of the layer. Soil-pile interaction is modeled by a bed of springs and dashpots (the springs representing soil stiffness, the dashpots energy loss due to radiation and hysteretic energy dissipation) connecting the pile to the free-field soil (Novak et al. 1978). For this analysis, the rod on dynamic Winkler foundation model has been applied to analyze the response of battered piles to lateral and vertical kinematic loads. The response of vertical pile to vertically propagation waves has also been the subject of research, (Kaynia and Kausel, 1982). With such an excitation a pile would undergo deflections over its entire length; hence the infinite beam model must be replaced with a finite one. For floating piles, the displacement of the tip is not known and it depends on the wavelength of the input motion. The study presented herein concentrates only on the response of floating pile, excited by a vertically incident P-wave and S-wave.

3. DISPLACEMENT OF SINGLE PILE

For flexible piles and low frequencies of incoming waves the pile follows closely the oscillating free field. The free-field and pile deformation are presented in Figs 3a, b.



a: P-wave b: SV-wave

Figure 3. Deforming and forces of batter pile under vertical and horizontal seismic excitation

Let corresponding soil displacement (subscript 'P' for P-wave and 'S' for SV-wave) of $u_{ff-P(S)}=u_{ff-P(S)}(z,t)$ that defined with $u_{ff-P(S)}=U_{ff-P(S)}(z)\exp(i\omega t)$ describe the base vertical (or horizontal) motion and $u_{ff0-P(S)}=U_{ff0-P(S)}\exp(i\omega t)$ the free-field vertical (or horizontal) displacement. The vertical base and free field motion are resultant to two vertical components that its displacements along the local horizontal and vertical pile axes. The vertical and horizontal components of ground motion are $u_{ff-P(S)v}(z)$ and $u_{ff-P(S)h}(z)$ respectively. Also for free field motion, vertical component is $u_{ff0-P(S)v}$ and horizontal component is $u_{ff0-P(S)h}$. These ground motion and free field displacement are reason for displacement batter pile. The vertical and horizontal components of pile displacement at depth are $u_{pf-P(S)v}(z)$, $u_{pf-P(S)h}(z)$ and at free field are $u_{pf0-P(S)v}$, $u_{pf0-P(S)h}$ respectively. The resultant these components on the global vertical and horizontal pile axes at depth are $U_{pf-P(S)V}(z)$, $U_{pf-P(S)H}(z)$ and at free field are $U_{pf0-P(S)V}$, $U_{pf0-P(S)H}$ respectively.

4. MODEL EQUATIONS

4.1. VERTICAL DIRECTION

The equilibrium of vertical forces acting on the elementary pile segment of Fig. 2 is written as (Mylonakis and Gazetas, 2002)

$$\frac{\partial P_V}{\partial z}+m\frac{\partial u_{pf-P(S)v}}{\partial t^2}+\left(k_V+ic_V\omega\right)\left(u_{pf-P(S)v}-u_{ff-P(S)v}\right)=0 \tag{1}$$

$$P_V=P_V(z)\Rightarrow u_{pf-P(S)v}=u_{pf-P(S)v}(z,t) \quad \& \quad u_{ff-P(S)v}=u_{ff-P(S)v}(z,t) \tag{2}$$

$P_V=P_V(z)$ denote axial force; k_V and c_V moduli of the distributed vertical soil springs and dashpots; m=pile mass per unit pile length. Restricting the analysis to harmonic vibrations, $u_{pf-P(S)v}(z,t)=u_{pf-P(S)v}(z)\exp(i\omega t)$, $u_{ff-P(S)v}(z,t)=u_{ff-P(S)v}(z)\exp(i\omega t)$, Eq. (1) yields the governing differential equation

$$\frac{\partial^2 u_{pf-P(S)v}}{\partial z^2}-\lambda_V{}^2 u_{pf-P(S)v}=-\frac{k_V+ic_V\omega}{E_P A_P}u_{ff-P(S)v} \tag{3}$$

In which (Gazetas et al. 1992)

$$k_V\cong 0.6E_s\left(1+0.5\sqrt{a_0\left(\frac{V_S}{V_P}\right)}\right), \qquad c_V\cong 1.2\left(a_0\left(\frac{V_S}{V_P}\right)\right)^{-1/4}\pi d\rho_s V_s+2\beta\left(\frac{k_V}{\omega}\right) \tag{4}$$

And

$$\lambda_V = \left[\frac{k_V + ic_V\omega - m\omega^2}{E_P A_P} \right]^{0.5} \quad (5)$$

λ_V is a complex wave number (units=length^{-1}) pertaining to the attenuation of pile response with depth (Novak and Aboul-Ella, 1978). The free-field soil motion, $u_{ff-P(S)v}(z)$, can be cast in the form

$$u_{ff-Pv}(z) = u_{ff0-Pv} \cos q_{P(S)}^{\ *} z \quad (6)$$

which corresponds to a standing wave satisfying the stress-free condition at the soil surface. In the above equation, u_{ff0-Pv}=vibration amplitude at the surface, while $q_{p(s)}$*=complex wave number

$$q_{P(S)}^{\ *} = \frac{\omega}{V^{*}_{P(S)}} \quad (7)$$

$$V^{*}_{P(S)} = V_{P(S)} \sqrt{(1+2i\beta)} \quad (8)$$

In which $V_{P(S)}$* complex propagation velocity of damped compressional waves (shear wave) in the soil medium (i=√-1). For locally vertical axes of pile, the fore of top pile is zero and for tie pile, the arguments of Novak and Aboul-Ella, 1978; assume that the pile toe acts as a rigid disk on the surface of a homogeneous elastic stratum of thickness equal to the distance from the pile toe to bedrock. Accordingly, the pertinent boundary condition is

$$P_V\big|_{z=L} = K_{bV}(u_{pf-P(S)v} - u_{ff-P(S)v})\big|_{z=L} \quad (9)$$

where K_{bV}=complex dynamic impedance of the disk. The solution used by Novak and Aboul-Ella, 1978) is adopted herein. Also, the distributed frequency dependent springs and dashpots k and c can be taken from available solutions by Novak et al., 1978; Roesset, 1980; and others. This paper utilizes the finite-element-based springs and dashpots of Gazetas et al. 1992;

$$K_{bV} = \frac{E_s(1+2i\beta)d}{1-\nu_s^2} + i\omega\pi\left(\frac{d}{2}\right)^2 \rho_s V_{La} \quad (10)$$

Where V_{La}=so-called "Lysmer's analog" wave velocity.

$$V_{La} = \frac{3.4}{\pi(1-\upsilon_s)} V_s \quad (11)$$

Enforcing the boundary condition in Eq. (9) and considering stress-free conditions at the pile head, the solution to Eq. (3) is obtained as

$$u_{pf-P(S)v}(z) = u_{ff0-P(S)v}\left[\frac{\Theta_{P(S)} q_{P(S)}^{\ *} \sin q_{P(S)}^{\ *} L + \Omega\lambda_V (1-\Theta_{P(S)}) \cos q^* L}{\lambda_V (\Omega \cosh \lambda_V L + \sinh \lambda_V L) \cos q_{P(S)}^{\ *} L} \times \cosh \lambda_V z + \Theta_{P(S)} \cos q_{P(S)}^{\ *} z \right] \quad (12)$$

where Ω and $\Theta_{P(S)}$=dimensionless factors:

$$\Omega = \frac{K_b}{E_P A_P \lambda_V} \quad (13)$$

$$\Theta_{P(S)} = \frac{k_V + ic_V\omega}{E_P A_P \left(q_{P(S)}^{*^2} + \lambda_V^{\ 2}\right)} \quad (14)$$

Of these factors, Ω expresses dimensionless pile toe stiffness while $\Theta_{P(S)}$ pertains to a particular solution of Eq. (3). Also, $\Theta_{P(S)}$ is related to the response of an infinitely long pile.

VERTICAL KINEMATIC RESPONSE FACTOR

To develop insight into the nature of the solution, it is instructive to introduce the vertical kinematic response factor

$$I_{p(s)v} = \frac{u_{pf-P(S)v}}{u_{ff0-P(S)v}} \tag{15}$$

which relates the vibration amplitude at the pile head ($u_{pf0-P(S)v}$) to that at the surface of the free-field soil ($u_{ff0-P(S)v}$). Without soil-pile interaction, $u_{pf-P(S)v}$ would be equal to $u_{ff-P(S)v}$ and $I_{p(s)v}$ equal to one. In reality, however, u_{ff-pv} and u_{pf-pv} are unequal in both amplitude and phase, and, thereby, $I_{p(s)v}$ is generally complex. Due to space limitations, emphasis will be given to the amplitude of $I_{p(s)v}$, which suffices for most practical applications, from Eqs. (12) and (15) $I_{p(s)v}$ is obtained as

$$I_{p(s)v} = \frac{\Theta q^* \sin q^* L + \Omega\lambda_V (1-\Theta)\cos q^* L}{\lambda_V (\Omega \cosh \lambda_V L + \sinh \lambda_V L)\cos q^* L} + \Theta \tag{16}$$

In which λ, $q_{p(s)}$*, $\Theta_{p(s)}$, and Ω are given by Eqs. (5), (7), (13) and (14), respectively. Some special cases are examined below. For an end-bearing pile, $\Omega \rightarrow \infty$, and for the particular case of a pile which is completely free of reaction at the toe (termed "fully floating pile") Ω vanishes, also finally in this paper, for an infinitely long pile, $L \rightarrow \infty$, the hyperbolic functions in the denominators of the preceding equations become very large; all equations converge to the remarkably simple expression, thus, Eq. (16) simplifies to:

$$I_{p(s)v} = \Theta_{p(s)} \tag{17}$$

4.2. HORIZONTAL DIRECTION

The dynamic equilibrium of horizontal forces under steady state condition acting on the elementary pile segment of Fig. 1 with $u_{pf-P(S)h}(z,t)=u_{pf-P(S)h}(z)\exp(i\omega t)$, $u_{ff-P(S)h}(z,t)=u_{ff-P(S)h}(z)\exp(i\omega t)$ is written as (Makris and Gazetas, 1992)

$$E_P I_P \frac{d^4 u_{pf-P(S)h}}{dz^4} + m\omega^2 u_{pf-P(S)h} + (k_H + i\omega c_H)(u_{pf-P(S)h} - u_{ff-P(S)h}) = 0 \tag{18}$$

$$u_{pf-P(S)h} = u_{pf-P(S)v}(z,t) \quad \& \quad u_{ff-P(S)h} = u_{ff-P(S)v}(z,t) \tag{19}$$

In which

$$k_H \cong 1.2 E_s \quad \& \quad c_H \cong 6 a_0^{-1/4} d\rho_s V_s + 2\beta\left(\frac{k_H}{\omega}\right) \tag{20}$$

For the linear hysteretic soil assumed herein, the total free-field displacement, $u_{ff-Sh}(z)$, is

$$u_{ff-P(S)h}(z) = u_{ff0-P(S)h} \cos \delta_{P(S)}{}^* z \tag{21}$$

which corresponds to a standing wave satisfying the stress-free condition at the soil surface. In the above equation, u_{ff0-Sh}=vibration amplitude at the surface, while $\delta_{P(S)}$*=complex wave number

$$\delta_{P(S)}{}^* = \frac{\omega}{V^*{}_{P(S)}} \tag{22}$$

The total solution of this equation is the sum of the homogeneous and a particular solution. After solution of this equation, for locally horizontal axes of pile boundary conditions are that the moment and shear force on the top and toe pile are zero. For most practical cases of interest, the participation of the homogeneous solution is not as important to the total solution and can indeed be neglected. Accordingly, in the sequel, the pile deflection is approximated by (particular solution)

$$u_{pf-P(S)h}(z) = u_{ff0-P(S)h}(\Gamma_{P(S)}) \tag{23}$$

In which

$$\Gamma_{P(S)} = \frac{k_H + i\omega c_H}{E_P I_P \delta^{*4} + k_H - m\omega^2 + i\omega c_H} = \frac{k_H + i\omega c_H}{E_P I_P \delta_{P(S)}{}^{*4} + 4\lambda_H^4} \tag{24}$$

And

$$\lambda_H = \left[\frac{k_H - m\omega^2 + i\omega c_H}{4E_P I_P}\right]^{0.25} \tag{25}$$

HORIZONTAL KINEMATIC RESPONSE FACTOR

To develop insight into the nature of the solution, it is instructive to introduce the horizontal kinematic response factor

$$I_{p(s)h} = \frac{u_{pf-Ph}}{u_{ff0-Ph}} \Rightarrow I_{p(s)h} = \Gamma_{p(s)} \tag{26}$$

After solution of differential equations of local vertical and horizontal axes of batter pile and determination global horizontal and vertical deformation of batter pile ($U_{pf0-P(s)H}$, $U_{pf0-P(s)V}$) the total kinematic response factors are expressed with

$$, I_{P(S)H} = \frac{U_{pf-P(S)H}}{u_{ff0-P(S)}} \quad I_{P(S)V} = \frac{U_{pf-PV}}{u_{ff0-P(S)}} \tag{27}$$

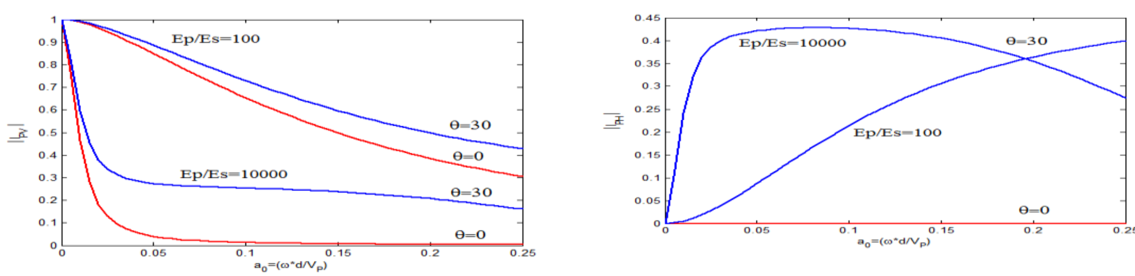
5. COMBINATION OF P-WAVE AND SV-WAVE ON THE KINEMATIC RESPONSE FACTOR OF PILE-SOIL

With definition of above relations, the combination of kinematic response factor of pile-soil under two waves is:

$$\begin{bmatrix} U_{pf_0-V} = U_{pf_0-SV} + U_{pf_0-PV} \\ U_{pf_0-H} = U_{pf_0-SH} + U_{pf_0-PH} \end{bmatrix} = \begin{bmatrix} u_{ff_0-S} \times I_{SV} + u_{ff_0-P} \times I_{PV} \\ u_{ff_0-S} \times I_{SH} + u_{ff_0-P} \times I_{PH} \end{bmatrix} = \begin{bmatrix} I_{SV} & I_{PV} \\ I_{SH} & I_{PH} \end{bmatrix} \times \begin{bmatrix} u_{ff_0-S} \\ u_{ff_0-P} \end{bmatrix} \tag{28}$$

6. COMPARISON OF RESULTS

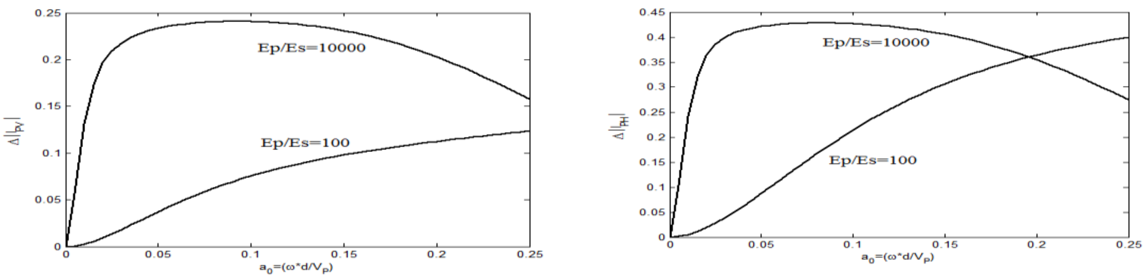
With definition of the total kinematic response factors under vertical wave are expressed with components of matrix are explained in Eq. (28), these values versus dimensionless frequency $a_0=\omega d/V_P$ are shown in Fig 4. Values I_{PH} for θ=0 are equal result with values of solution Makris and Gazetas, 1992; For vertical and horizontal kinematic response factors are



Vertical (I_{PV}) Horizontal (I_{PH})

Fig 4. Normalized kinematic seismic response of batter pile versus $a_0=\omega d/V_P$ with different batter angle (θ) and E_P/E_S (ρ_p/ρ_s=1.60, ν=0.4, β=0.05, L/d→∞) under P-wave

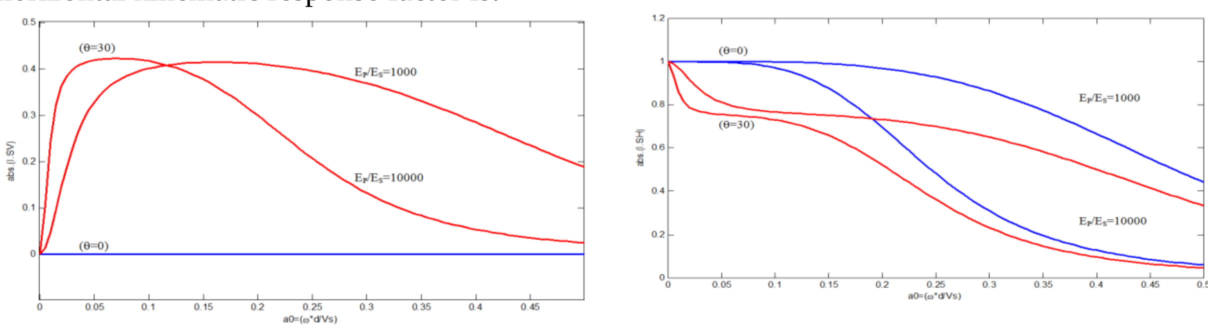
Also difference between values kinematic response factor for θ=30 and θ=0 for determination influence batter angle on kinematic response factor versus $a_0=\omega d/V_P$ are illustrated on Fig. 5.



Difference of vertical (ΔI_{PV}) Difference of vertical (ΔI_{PH})

Fig 5. Difference between values kinematic seismic response of batter pile versus $a_0=\omega d/V_P$ with different E_P/E_S (ρ_p/ρ_s=1.60, ν=0.4, β=0.05, L/d$\rightarrow\infty$) under P-wave

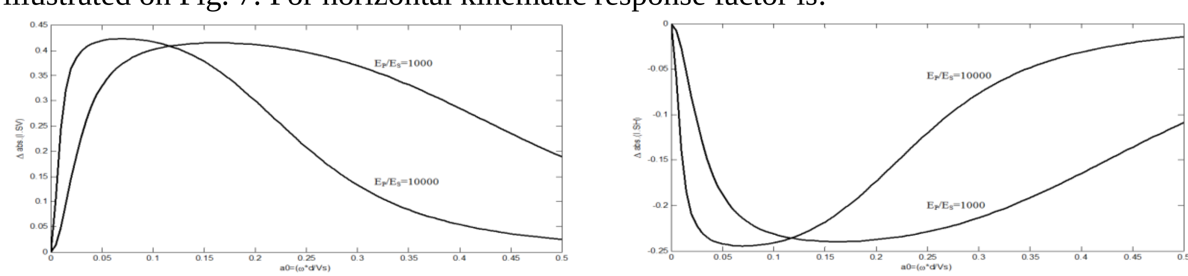
Components of matrix expressed in Eq. (28) under horizontal wave, the values of kinematic response factors values versus dimensionless frequency $a_0=\omega d/V_S$ are shown in Fig. 6.Values I_{SH} for θ=0 are equal result with values of solution Makris and Gazetas, 1992. For vertical and horizontal kinematic response factor is:



Vertical (I_{SV}) Horizontal (I_{SH})

Fig 6. Normalized kinematic seismic response of batter pile versus $a_0=\omega d/V_S$ with different batter angle (θ) and E_P/E_S (ρ_p/ρ_s=1.42, ν=0.4, β=0.05, L/d$\rightarrow\infty$) under SV-wave

Also difference between values kinematic response factor for θ=30 and θ=0 for determination influence batter angle on kinematic response factor versus dimensionless frequency are illustrated on Fig. 7. For horizontal kinematic response factor is:



Difference of vertical (ΔI_{SV}) Difference of vertical (ΔI_{SH})

Fig. 7. Difference between values kinematic seismic response of batter pile versus $a_0=\omega d/V_S$ with different E_P/E_S (ρ_p/ρ_s=1.42, ν=0.4, β=0.05, L/d$\rightarrow\infty$) under SV-wave

7. CONCLUSIONS

A general method has been developed to compute total kinematic response factor under vertical and horizontal wave for batter pile under different batter angle. Determination kinematic response factor in this paper are expressed by simple closed-form expression with a dynamic Winkler model. In values are expression with variation of ρ_p/ρ_s, ν, β, L/d and E_P/E_S versus dimensionless frequency $a_0=\omega d/V_P$ and $a_0=\omega d/V_S$

For P-wave, vertical kinematic response factor with increases a_0, E_P/E_S unlike of batter angle (θ) is decreased. Also horizontal kinematic response factor at vertical pile under vertical excitation is zero and at any batter angle, with increases a_0 for lower E_P/E_S is growth and at preliminary dimensionless frequency, that its values depended with values ρ_p/ρ_s, ν, β, L/d, with increases E_P/E_S are increased and at continuance are decreased. Also these values of horizontal kinematic seismic response with increase batter angle are increased. Thus batter pile under vertical seismic excitation for horizontal manner is not better than vertical.

But, for SV-wave, horizontal kinematic response factor with increases a_0, E_P/E_S and batter angle (θ) is decreased. Also vertical kinematic response factor at vertical pile under horizontal excitation is zero and at any batter angle, with increases a_0 is reduced but at preliminary dimensionless frequency that its values depended with values ρ_p/ρ_s, ν, β, L/d with increases E_P/E_S are increased and at continuance are decreased. Also these value inverses of horizontal kinematic seismic response with increase batter angle are decreased. Thus batter pile under horizontal seismic excitation for horizontal manner is better than vertical.

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